Research Announcement:
COMMON FIXED POINT THEOREMS

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Definition. (Takahashi) Let X be a metric space and I be closed unit interval. A mapping \( W : X \times X \times I \rightarrow X \) is said to be a convex structure on X if for all \( x, y \in X \) and \( \lambda \in I \) the following condition is satisfied

\[
d(u, W(x, y; \lambda)) \leq \lambda d(u, x) + (1-\lambda)d(u, y)
\]

for all \( u \) in X. A metric space with a convex structure is called a convex metric space.

**Theorem 1.** Let \( K \) be a nonempty compact convex subset of a convex metric space \( X \). If \( S \) is a left reversible semigroup of nonexpansive mappings of \( K \) into itself then \( K \) contains a common fixed point of \( S \).

**Theorem 2.** Let \( X \) be a compact metric space and \( G : X \rightarrow X \) be a linearly ordered semigroup of mappings. Suppose \( G \) has diminishing orbital diameter and there exists \( g \in G \) with \( g \neq I \) such that

(i) \( G \) is continuous mapping with diminishing orbital diameter,

(ii) \( G \) is Archimedean at \( g \).

Then \( G \) has a common fixed point.

**Theorem 3.** Let \( X \) be a convex metric space having property (C) and \( H \) be a closed convex subset of \( X \). Let \( K \) be a bounded, closed convex subset of \( H \) with normal structure. 
If $T: K \rightarrow H$ is nonexpansive and if $T: \partial_H K + K$ ($\partial_H K$ is the relative boundary of $H \cap K$ in $H$), then $T$ has a fixed point in $K$.

Also we prove a common fixed point theorem for commuting linearly ordered semigroup of nonexpansive mappings having convex diminishing orbital diameter.

Theorem 1 generalizes results of De Marr [1], Mitchell [4] and Takahashi [5,6]. Theorem 2 and 3 extend the results of Kirk [2], [3] respectively.

References


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