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# QUESTIONS ON HOMEOMORPHISM GROUPS OF CHAINABLE AND HOMOGENEOUS CONTINUA

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# QUESTIONS ON HOMEOMORPHISM GROUPS OF CHAINABLE AND HOMOGENEOUS CONTINUA

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The following theorem is likely to be of importance in the solution of the problems posed below.

*Theorem* (Effros). *Let* X *be a homogeneous metric continuum. Then for every* $\varepsilon > 0$, *there exists* $\delta > 0$ *such that if* $d(x,y) < \delta$, *then there is a homeomorphism* $h: X \twoheadrightarrow Y$ *such that* $d(h, id) < \varepsilon$ *and* $h(x) = y$.

In [2], we began a study of the topological structure, in particular dimension properties, of homeomorphism groups of various continua. In particular, it is shown that the groups of homeomorphisms of locally-setwise-homogeneous continua are non-zero dimensional, and, in fact, contain the infinite product of non-zero dimensional subgroups. Such continua include the Sierpinski universal plane curve and the Menger universal curve. The homeomorphism groups of these two continua are also totally disconnected, and it is still an *open question* to determine what the dimension is. Examples $M_n$ are also constructed in [2], with the property that $G(M_n)$ is topologically and algebraically the product of n 1-dimensional groups. It is still *unknown* what their dimension is, too.

Here we list some questions about the homeomorphism groups of the pseudo arc and other homogeneous continua. These questions were raised by the author at the University of Texas Summer '80 Topology Conference, held in Austin, Texas.

Let P be the pseudo arc, and let X be any homogeneous metric continuum. Let H(X) denote the group of all homeomorphisms of X onto itself. It is well known and easy to see that H(P) contains no arcs: For any such arc is a homotopy $\{h_t\}$ of P, and if $\{x\} \times I$ is a track of the homotopy such that $h_1(x) \neq x$, then $\cup\{h_t(x)\}_{t \in I}$ is a subcontinuum of P which is a continuous image of an arc, and therefore locally connected. But P contains no non-degenerate locally connected continua. Thus, we raise the following:

*Questions.*

(1) Is H(P) totally disconnected? 0-dimensional? infinite dimensional?

(2) Does H(P) contain a pseudo arc? an infinite product of pseudo arcs? (Wayne Lewis [7] has just answered this question in the negative, by showing that H(P) contains no non-degenerate subcontinua.)

(3) Is H(P) connected? If not, does it contain a non-degenerate component?

(4) Let G denote the subgroup of H keeping every composant invariant. Then G is normal in H. Is G minimal normal? (See [1,3,8].) What is the (non-identity) minimal normal subgroup? Is G generated by those homeomorphisms supported on small open sets? (See [5].)

(5) Let X be any homogeneous metric continuum. Is H(X) non-zero dimensional? infinite dimensional?

*Remark.* It has recently been shown by Wayne Lewis [6] that the pseudo arc admits p-adic Cantor group actions, as well as period n homeomorphisms for all n.

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