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HOMOGENEOUS CONTINUA

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Fundamenta Mathematica was the first journal devoted to set theory. Lebesgue, among others, applauded the effort but worried that a dearth of publishable work might doom the enterprise [10]. Perhaps it was to avoid this calamity and to prime the pump that the editors included a list of questions at the end of each volume.

The first question in the first volume in 1920 was answered almost immediately, but the second was a dilly. Knaster and Kuratowski [25] asked if each homogeneous, plane continuum must be a simple closed curve. Mazurkiewicz [33] proved in 1924 that the answer is yes provided the continuum is locally connected. This was the only significant progress on the problem for over a quarter century, even though the problem did not sit on the back burner.

In 1948, R. H. Bing [3] proved that the pseudo-arc is homogeneous. This remarkable result initiated a spate of activity on the problem. In some sense, this period of intense activity was concluded in 1961 by another paper [6] of Bing, in which he showed that the answer to the question is yes provided the continuum contains an arc. This could be called the classical or planar period in the study of homogeneous continua. Although homogeneous continua in general were also investigated, the predominant results

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continued to be spawned by the original question of Knaster and Kuratowski.

Later in the decade two additional and important results were obtained, results that concluded the classical period. In 1968, L. Fearnley [15] and the author [46] independently proved that the pseudo-circle is not homogeneous, and in 1969, at the Auburn Topology Conference, F. B. Jones [22] announced that indecomposable, homogeneous, plane continua must be hereditarily indecomposable. The pseudo-circle, defined by Bing [4] almost 20 years earlier, had emerged as the leading candidate for a new homogeneous continuum. The fact that it is not homogeneous suggested that new homogeneous continua in the plane would be hard to come by.

The proofs of these two results told the tale on the state of the art at that time. Jones never wrote up his proof--he told me once that it would have been so complicated that he feared no one would read it. In the same vein, I felt that the ideas in the proof of the nonhomogeneity of the pseudo-circle should extend to some other separating plane continua, but the details were formidable, and I was never tempted more than briefly to attack them. The reader should recall that in those days, to prove the pseudo-circle nonhomogeneous, certain points x and y were precisely prescribed, and it was shown that no homeomorphism of the continuum could move the point x to the point y.

Clearly, new techniques were needed if the study of the homogeneous continua were to remain a viable field. The most important such technique was already available, although we didn't know it. In 1965, E. G. Effros [14] proved an important result about Polish transformation groups. When applied to the homeomorphism group of a homogeneous continuum, it yields a powerful and effective tool.

G. Ungar [50] was the first to apply the Effros result to continua; with it, he showed that 2-homogeneity implies local connectivity. It is significant that this is a nonplanar result (C. E. Burgess, one of the pioneers in the study of homogeneous continua, had already shown the result in the plane and had raised the question in general [11].)

In 1975, then, the study of homogeneous continua entered its current state--the modern or nonplanar period--a second period of intense activity, marked by extensive use of the Effros result and punctuated by the introduction of other new techniques as well.

1. Definitions and Goals of the Paper

The goals of this paper are to summarize the present state of knowledge of homogeneous continua, to present a possible classification of all homogeneous continua, to ask some questions whose answers are important in obtaining further progress, and to mention some of the new techniques currently being used in the investigation of these continua

The classification scheme rests on the cornerstone of Jones' Aposyndetic Decomposition Theorem. We present the scheme first for plane continua, then for curves, and finally, for continua of dimension greater than one.

A continuum is a compact, connected, nonvoid metric space. A curve is a one-dimensional continuum.

A space X is homogeneous if, for each pair of points x and y of X, there exists a homeomorphism of (X,x) onto (X,y).

A continuum X is decomposable if it is the union of two of its proper subcontinua; otherwise, X is indecomposable. A continuum is hereditarily indecomposable if it does not contain a decomposable continuum.

A pseudo-arc is a chainable, hereditarily indecomposable continuum.

The notion of an aposyndetic continuum is crucial to the investigation of homogeneous continua. For the purposes of this survey, however, it is sufficient to consider aposyndesis as a weak form of local connectivity and to know that the following implications hold, and that none are reversible: locally connected $\Rightarrow$ aposyndetic $\Rightarrow$ decomposable.

2. A Classification of Homogeneous, Plane Continua

In 1949, Jones [23] proved that an aposyndetic, homogeneous, plane continuum is either a point or a simple closed curve.

In 1951, Jones [20] made the first use of decompositions of homogeneous continua by showing that a nonseparating, homogeneous, plane continuum must be indecomposable. In 1954, he divided homogeneous plane continua into three types:

(A) Nonseparating (hence indecomposable).

(B) Separating and decomposable.

(C) Separating and indecomposable.

Furthermore he showed [21] that each Type (B) continuum is a "circle" of Type (A) continua.

The author [43] proved that the set of Type (C) continua is empty. C. L. Hagopian [16 and 17] proved that Type (A) continua are hereditarily indecomposable.

There are, at present, four known homogeneous, plane continua. The point and the pseudo-arc are Type (A) continua; the circle and the circle of pseudo-arcs are Type (B) continua. An affirmative answer to the following question of Jones would imply that these four are the only homogeneous, plane continua.

Question 1. Is each non-degenerate homogeneous, nonseparating, plane continuum a pseudo-arc?

L. G. Oversteegen and E. D. Tymchatyn [36 and 37] have recently taken a long stride toward solving the problem by showing that each Type (A) continuum has span zero and is a continuous image of the pseudo-arc.

3. Homogeneous Curves Outside the Plane

Homogeneous, nonplanar curves are a more yeasty mixture. There exists, for instance, a collection of cardinality c of solenoids. A solenoid is defined as an inverse limit of circles with covering maps as the bonding maps. Each solenoid is an indecomposable continuum as well as an abelian topological group. Hence each solenoid is an indecomposable, homogeneous continuum with nontrivial cohomology.

If $f: S \to S^1$ is the projection map of the solenoid S onto the factor space S^1, then f is a morphism of topological groups with kernel a topological group G whose underlying space is a Cantor set. The collection $S = (S,f,S^1,G)$ is a principal fiber bundle.

In 1958, R. D. Anderson [1] showed that Menger universal curve (the so-called "Swiss Cheese Space") is homogeneous, and that the circle and the Menger curve are the only homogeneous, locally connected curves.

In 1961, J. H. Case [12] constructed a new homogeneous curve as an inverse limit of universal curves and double-covering maps. Case's construction was quite complicated, and in 1982 the author [47] provided a simpler, geometric construction of similar continua and then [45] a bundle-theoretic construction of such spaces. These continua are simply the total spaces of bundles induced from solenoid bundles by a retraction of the universal curve onto a "core" circle.

It can be proved from these constructions that there are c such continua (one for each solenoid), that each is aposyndetic but not locally connected, and that none is arcwise-connected, hereditarily decomposable, or pointed-one-movable.

In 1983, P. Minc and the author [34] constructed even more homogeneous, aposyndetic curves. The geometric idea is to spin the Menger curve around several of its holes at the same time. Each finite sequence of solenoids $S_1,\cdots,S_n$ determines one of these continua M. If M' is another such continuum determined by the sequence $S_1',\cdots,S_m'$ and if M' is homeomorphic to M, then $n = m$ and S_i is homeomorphic to S_i', for some reindexing.

4. Six Types of Homogeneous Curves

We propose here a classification of homogeneous curves by dividing them into six types.

Type 1. *Locally connected.* The universal curve and the circle are the only ones [1], so this type is completely understood.

Type 2. *Aposyndetic but not locally connected.* The examples of Case, Rogers, and Minc and Rogers are the only examples known. All these examples can be obtained as inverse limits of universal curves and covering maps. All of them can also be obtained as total spaces of Cantor set bundles over the Menger curve. This part of the theory is in ferment, and we ask the following questions:

Question 2. Is each Type 2 curve a bundle over the universal curve with Cantor sets as the fibers?

Question 3. Is each Type 2 curve an inverse limit of universal curves and maps? universal curves and fibrations? universal curves and covering maps?

Question 4. Does each Type 2 curve contain an arc?

Question 5. Does each Type 2 curve retract onto a solenoid?

Type 3. *Decomposable but not aposyndetic.* The crux of the matter here is the Jones' Aposyndetic Decomposition Theorem [21] as improved in [48].

Theorem. Each Type 3 curve admits a continuous decomposition into Type 6 curves such that the quotient space is a Type 1 or Type 2 curve.

Jones' theorem tells us, in one sense, not to worry about Type 3 curves until we know enough about Type 1, Type 2, and Type 6 curves.

The (as yet mysterious to the reader) Type 6 curves contain only one known element--the pseudo-arc. In view of Jones' theorem, it is natural to ask if each Type 1 or Type 2 curve can be realized as a decomposition of a Type 3 curve into pseudo-arcs. More generally, there is the problem, given a homogeneous curve X, of "blowing up" its points into pseudo-arcs to obtain a homogeneous curve $\tilde{X}$.

Bing and Jones [9] solved this problem for the circle. It follows from their construction that, to any finite, connected graph G, there corresponds a curve $\tilde{G}$ and a decomposition of $\tilde{G}$ into pseudo-arcs with quotient space G.

The author [40] solved this problem for solenoids. The idea in that paper is to express a curve X as an inverse limit of graphs (G,g), use Bing-Jones to blow up the graphs G to graphs of pseudo-arcs $\tilde{G}$, and obtain $\tilde{X}$ as an inverse limit of $(\tilde{G},\tilde{g})$ such that $\tilde{X}$ admits a continuous decomposition into pseudo-arcs with quotient space X.

The problem then is to show that X homogeneous implies $\tilde{X}$ homogeneous. In the case (such as for the solenoids) that X is homogeneous by homeomorphisms induced by commuting diagrams of maps on the inverse sequence (G,g), the desired homeomorphisms on $\tilde{X}$ can be obtained by a straightforward lifting process [40].

But it is not known that there are always enough induced homeomorphisms on X to do the job, and in fact, it seems unlikely that this is always so. In the absence of induced homeomorphisms, one must fall back to Mioduszewski's ε-commutative diagrams [35], and then appears the sticky problem of whether the lift to $(\tilde{G},\tilde{g})$ of an "almost commutative" diagram involving (G,g) is still "almost commutative enough." Fortunately, by a careful use of the Bing-Jones paper, Wayne Lewis [30] has proved that this is indeed possible, and that hence, for each homogeneous curve X, there is a homogeneous curve $\tilde{X}$ that admits a continuous decomposition into pseudo-arcs with quotient space X.

Incidentally, the problem of replacing a map between inverse limit spaces by a map induced from commuting diagrams on the inverse sequences is an important problem in continua theory. One would wish the the induced map to have any desirable property (such as being a homeomorphism taking the point x to the point y) possessed by the original map. More about this possibility and its limitations is needed.

Type 4. *Indecomposable and cyclic.* A curve is said to be cyclic if its first Čech cohomology group with integer coefficients does not vanish; otherwise it is acyclic. The solenoids and the solenoids of pseudo-arcs are the only such continua known.

Question 6. Does each Type 4 curve that is not a solenoid admit a continuous decomposition into Type 6 curves so that the resulting quotient space is a solenoid?

C. L. Hagopian [17] has shown that the answer is yes for atriodic curves.

Type 5. *Acyclic and not tree-like.* This territory has yet to be investigated intensively. Bing [7] has shown that acyclic plane curves are tree-like, so this is definitely a nonplanar problem.

Question 7. Is the set of Type 5 curves empty? Equivalently, does trivial cohomology imply trivial shape, for homogeneous curves?

Type 6. *Tree-like.* The pseudo-arc is still the only Type 6 curve known. To prove it is the only one seems fraught with difficulties.

Jones [20] has shown that a new example must be indecomposable. Lewis [17] has shown that a new example must be infinitely-branched and infinitely-junctioned and must contain a proper nondegenerate subcontinuum different from a pseudo-arc [28]. Hagopian [18] has shown that no example can contain an arc.

The author [44] has shown that any hereditarily indecomposable, homogeneous continuum must be a Type 6 curve. Oversteegen and Tymchatyn [38] have provided a new proof that the pseudo-arc is homogeneous. Bing [5] has shown that the pseudo-arc is the only chainable, homogeneous continuum.

In the next question, we summarize the questions that have been asked by different investigators in seeking further restrictions on Type 6 curves.

Question 8. Are Type 6 curves hereditarily indecomposable? pseudo-arcs? weakly chainable? hereditarily equivalent? Do they have span zero? the fixed-point property?

5. Homogeneous Continua of Higher Dimension

Homogeneous continua of dimension greater than one can be divided similarly into six types, but in general they form a rather intractible class with questions arising from varied sources.

Type 1. *Locally connected.* Closed n-manifolds (for $n > 1$), countable products of locally connected, homogeneous, nondegenerate continua, and the Hilbert cube are Type 1 continua. K. Kuperberg [26] has shown that certain mapping tori are Type 1 continua. Higher-dimensional analogues of the Menger curve may be homogeneous--they are discussed elsewhere in this conference.

Type 2. *Aposyndetic and not locally connected.* All non-trivial products of continua are aposyndetic. Hence any non-trivial countable product of homogeneous, nondegenerate continua one of whose factors is not locally connected is a Type 2 continuum. If M is a closed n-manifold (for $n > 1$) that admits a retraction onto a finite wedge of circles, then the bundle machines of [45] and [34] automatically provide an n-dimensional Type 2 continuum. Some have speculated that certain mapping tori are Type 2 continua.

Type 3. *Decomposable and not aposyndetic.* Again the Jones' Aposyndetic Decomposition Theorem comes into play, this time in its full generality [48].

Theorem. Each decomposable, homogeneous continuum admits a continuous decomposition into mutually homeomorphic, cell-like, indecomposable, homogeneous continua such that the quotient space is an aposyndetic, homogeneous continuum.

A continuum is said to be cell-like if it has the shape of a point. Note that neither points nor tree-like curves are excluded as the elements of the aposyndetic decomposition.

No Type 3 continuum is known, which suggests the following question:

Question 9. Is each decomposable, homogeneous continuum of dimension greater than one aposyndetic?

Answers to the following questions would strengthen the Decomposition Theorem as well as shed some light on Question 9.

Question 10. Must the elements of this aposyndetic decomposition be hereditarily indecomposable?

Question 11. Can this aposyndetic decomposition raise dimension? lower dimension?

The last question reveals an interface between homogeneous continua and the cell-like mapping problem, which asks if a cell-like mapping can raise dimension. This is such a pretty decomposition (the quotient mapping is even completely regular [47]) that perhaps some of our topological neighbors will give us a hand with the first part of Question 11.

Type 4 continua are the indecomposable and cyclic continua. Cyclic means that some (reduced) Čech cohomology group is nontrivial; otherwise the continuum is acyclic. Type 5 continua are the acyclic but not cell-like continua, and Type 6 continua are the cell-like ones.

No continuum of Type 4, 5, or 6 is known; these are really uncharted waters.

Question 12. Is each indecomposable, nondegenerate, homogeneous continuum one-dimensional?

The author [47] has shown that the answer is yes if the continuum is hereditarily indecomposable. Hagopian [17] has shown that any counterexample must contain a triod.

Some formidable obstacles lie in the path of a complete classification of high-dimensional homogeneous continua. For instance, Bing and Borsuk [8] conjectured in 1965 that an n-dimensional, homogeneous, compact ANR is an n-manifold, and they proved the conjecture true for $n = 1$ or 2. In 1980, however, W. Jakobsche [19] showed that the validity of the Bing-Borsuk conjecture for $n = 3$ implies the validity of the Poincaré Conjecture!

Consider also this baffling question from infinite-dimensional topology:

Question 13. Is each nondegenerate, homogeneous contractible continuum homeomorphic to the Hilbert cube?

6. A Decomposition Theorem

One of the most useful tools in studying homogeneous continua is decompositions. Here we state a version of the decomposition theorems of [41] for homogeneous curves.

Theorem. Let X *be a homogeneous curve, and let* H(X) *be its homeomorphism group. Let* $\mathcal{G}$ *be a partition of* X *into proper, nondegenerate continua such that* H(X) *respects* $\mathcal{G}$ *(this means that either* $h(G_1) = G_2$ *or* $h(G_1) \cap G_2 = \phi$*, for all* G_1 *and* G_2 *in* $\mathcal{G}$ *and all* h *in* H(X)*). Then*

1. $\mathcal{G}$ *is a continuous decomposition of* X*,*

2. *There is a continuum* G *such that each element of* $\mathcal{G}$ *is homeomorphic to* G*,*

3. G *is homogeneous, indecomposable, and tree-like,*

4. *The quotient space of this decomposition is a homogeneous curve.*

An affirmative answer to the next question would make decompositions an even more useful tool.

Question 14. Is G hereditarily indecomposable?

Here are some applications of this decomposition theorem. Let X denote a homogeneous curve.

Application 1. Suppose X contains an arc. Let $\mathcal{G}$ be the set whose elements are closures of arc components of X. One shows that $\mathcal{G}$ is a partition of X which H(X), of course, respects. Since no homogeneous, tree-like continuum can contain an arc [18], it follows that $\mathcal{G}$ contains only the one element X. Therefore, if the homogeneous curve contains an arc, then it contains a dense arc component [49].

Application 2. This is due to Wayne Lewis [28]. Suppose X is hereditarily indecomposable (this implies X is tree-like [44]), not a pseudo-arc, and contains a pseudo-arc. For each point x of X, let P_x be the closure of the union of all pseudo-arcs containing x. The continuum P_x is a pseudo-arc. Let $\mathcal{G} = \{P_x: x \in X\}$. The quotient space of this decomposition is a tree-like, homogeneous continuum containing no pseudo-arc.

Application 3. Suppose X is decomposable. Let L_x be the set that has as members the point x of X together with all points z of X such that M is not aposyndetic at z with respect to x. Let $\mathcal{G} = \{L_x: x \in X\}$. The resulting decomposition is the Jones' aposyndetic decomposition [21].

Application 4. There does not exist, for instance, a circle of solenoids. This means that no homogeneous curve admits a decomposition into solenoids such that the quotient space is a simple closed curve.

7. Potpourri

In this section we discuss two questions included just because they are interesting and not as well-studied as some of the classical questions. The first is due to Judy Kennedy and Wayne Lewis [31].

Question 15. Does each homogeneous continuum admit a nontrivial, primitively stable homeomorphism?

A homeomorphism is primitively stable if it is the identity when restricted to some nonempty open set.

Nontrivial means it is not the identity map of the continuum.

All known homogeneous continua admit such a homeomorphism. Kennedy [39] has shown that if the answer is yes, then 2-homogeneity implies n-homogeneity for all n. The author [49] has shown that if the answer is yes, then each homogeneous continuum that contains a simple triod also contains a simple closed curve. An affirmative answer also yields an alternative proof [49] to the Bellamy-Lum corollary [2] that an arcwise-connected, homogeneous curve contains a simple closed curve.

The second question, due to Ben Fitzpatrick, is somehow tempting.

Question 16. Is each homogeneous continuum bihomogeneous?

A continuum X is bihomogeneous if, given points x and y in X, there exists a homeomorphism h of X onto itself such that $h(x) = y$ and $h(y) = x$.

8. Completely Regular Maps

A surjective map $f: X \to Y$ between metric spaces is said to be completely regular if, for each $\varepsilon > 0$ and point y in Y, there exists a $\delta > 0$ such that $d(y,y') < \delta$ implies there exists a homeomorphism of $f^{-1}(y)$ onto $f^{-1}(y')$ moving no point as much as ε.

Projection maps of products are completely regular, and completely regular maps are open. In general, neither of the converse statements is true.

Dyer and Hamstrom introduced completely regular maps in [13], with the idea of showing that spaces on which certain open maps are defined are locally products. They considered, for instance, maps whose fibers are 2-spheres. Kim [24] has shown that their techniques, together with current knowledge about the homeomorphism group of a compact manifold, imply that each completely regular map with fibers homeomorphic to a compact manifold is locally trivial.

Completely regular maps arise naturally in the study of homogeneous continua, frequently as a consequence of using the Effros result. Moreover, these maps are often not locally trivial. Consider the following two theorems.

Theorem. In the Decomposition Theorem of Section 6, *the quotient map is completely regular.*

The second theorem is an immediate corollary of [9, Theorem 9].

Theorem. If f *is an open, surjective map between compacta with the property that each point inverse is a pseudo-arc, then* f *is completely regular.*

Completely regular maps have some special properties. We close the paper with two of them. The first, due to Mason and Wilson [32], is crucial in part of the proof of the Decomposition Theorem.

Theorem. If $f: X \to Y$ *is a completely regular, monotone map between curves, then* $f^{-1}(y)$ *is a tree-like continuum, for all* y *in* Y.

The second is due to Dyer and Hamstrom [13].

Theorem. Let f: X → Y *be a completely regular mapping between compacta. Let* $f^{-1}(y)$ *be homeomorphic to the compactum* M, *for all* y *in* Y. *Let* H(M) *be the homeomorphism group of* M. *Suppose* (1) *dim* Y $\leq$ n + 1, (2) H(M) *is* LC^n, *and* (3) $\Pi_i(H(M)) = 0$, $0 \leq i \leq n$. *Then* X *is homeomorphic to* Y × M.

The Dyer-Hamstrom result requires, for most applications, a well-behaved homeomorphism group H(M). If Y is a Cantor set, however, then n = -1 and conditions (2) and (3) are vacuously satisfied.

An application of this is the following. Call a compactum a Cantor set of pseudo-arcs if it admits an open map onto a Cantor set with pseudo-arcs as the fibers. Then we have an alternate proof of a result of Wayne Lewis [29]: Each Cantor set of pseudo-arcs is a product of a Cantor set and a pseudo-arc.

These ideas may have application again in continua theory.

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