TOPOLOGY PROCEEDINGS
Volume 11, 1986
Pages 225–238

http://topology.auburn.edu/tp/

THE FIXED REMAINDER PROPERTY FOR SELF HOMEOMORPHISMS OF ELSA CONTINUA

by

Marwan M. Awartani

Topology Proceedings

Web: http://topology.auburn.edu/tp/
Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA
E-mail: topolog@auburn.edu
ISSN: 0146-4124

THE FIXED REMAINDER PROPERTY FOR SELF HOMEOMORPHISMS OF ELSA CONTINUA

Marwan M. Awartani*

1. Introduction

An *Elsa Continuum* [2], denoted E-*continuum*, is a compactification of the ray (0,1], denote J, with a closed arc as remainder. An E-continuum E is said to have the *fixed remainder property* if every self homeomorphism of E is the identity on the remainder. Let FR denote the class of E-continua having the fixed remainder property. In [1] David Bellamy asked whether FR is nonempty. We answer this question in the affirmative. First (Theorem 3.7) we give sufficient conditions for FR. Then we prove (Theorem 4.1) that FR contains uncountably many topologically distinct E-continua. Necessary conditions for FR are given in corollary 5.2. The techniques used in this paper employ an intimate relation between a given E-continuum and the compactifications of its skeletons which are certain discrete subsets of E (see Definition 2.4). For related work see [2], [4], [5] and [6].

2. Preliminaries

Let E be an E-continuum. Then J(E) denotes the copy of J densely embedded in E and E* denotes the remainder $E\backslash J(E)$. In general if $A \subseteq E$, then $\bar{A}$ denotes the closure of A and $A^* = \bar{A}\backslash A$. If p and q are two points of J(E), then

*The author wishes to thank David Bellamy for valuable conversations on several aspects of Elsa continua.

(p,q), $[p,q]$ denote respectively the open, closed arc in $J(E)$ joining p and q. Let $d(p,q)$ denote the diameter of $[p,q]$. I denotes the closed unit interval $[0,1]$ and N denotes the set of positive integers.

Definition 2.1. Let E be an E-continuum. The *type* of E, denoted T(E), is the set of points p in E* having the following property:

There exists a sequence $\{[p_i,q_i]\}$ of arcs in J(E) such that p is an endpoint of the arc $\lim\{[p_i,q_i]\} \subseteq E^*$ and $\lim\{p_i\} = \lim\{q_i\} \neq p$. The convergence of $\{[p_i,q_i]\}$ is taken with respect to the Hausdorff metric on the hyperspace 2^E of closed subsets of E [3, p. 1].

The following theorem is a direct consequence of Definition 2.1:

Theorem 2.2. T(E) *is a topological invariant of* E.

Combining 3.1 of [2] and 2.3 of [4] we obtain the following:

Theorem 2.3. *Let* E *be an* E-*continuum. Then* E *can be embedded in the plane as the closure of the graph of a function* $f: J \to I$ *satisfying the following conditions:*

i) *There exists a sequence* V(E) *of points in* J(E) *such that each point of* V(E) *is a strict local maximum or a strict local minimum of* f. *Moreover* f *is linear between consecutive points of* V(E).

ii) $(V(E))^* = T(E)$, *the type of* E.

Henceforth, all E-continua are assumed to be already embedded in the plane in the particular manner described in the above theorem. Hence J(E) is the graph of the function f described above and $E^* = I$.

We linearly order points of J(E) by their first coordinate and points of E^* by their second coordinate. Let π_1 and π_2 denote the projections on the respective coordinates.

Definition 2.4. Let E be an E-continuum and let $V(E) = v_1, v_2, v_3, \cdots$. Then a *skeleton* of E is any subset of V(E) of the form $\{v_i\}_{i \geq i_o}$ for some $i_o \in N$.

3. Sufficient Conditions for FR

Definition 3.1. E is said to be *separated* if the seuqence $\{d(v,w)$: v,w are adjacent members of $V(E)\}$ is bounded away from 0.

Definition 3.2. A sequence $\{u_i\}$ of points in J(E) is said to be a V-*sequence* if there exists a subsequence $\{v_{k_i}\}$ of V(E) such that $\{d(v_{k_i}, u_i)\}$ converges to 0.

Lemma 3.3. *Let* E_1 *and* E_2 *be separated* E-*continua and let* $h: E_1 \to E_2$ *be a homeomorphism. If* $\{u_i\}$ *is a* V-*sequence in* E_1, *then* $h(u_i)$ *is a* V-*sequence in* E_2.

Proof. Due to compactness considerations, it suffices to consider the case where $\{u_i\}$ is a convergent sequence. Let $U = \lim \{u_i\}$. By Definition 3.2, there exists a subsequence $\{v_{k_i}\}$ of $V(E_1)$ such that $\{d(u_i, v_{k_i})\}$ converges to 0. Hence $\lim \{v_{k_i}\} = u$. We may assume without loss of generality that each v_{k_i} is a local minimum of $J(E)$. The case where the v_{k_i}'s are local maxima can be treated similarly.

Suppose that $\{h(u_i)\}$ is not a V-sequence in E_2. Then we may assume without loss of generality that $h(u_i) \in (t_{k_i}, t_{k_i+1})$, where t_{k_i} and t_{k_i+1} are consecutive elements of $V(E_2)$, and where the sequences $\{d(t_{k_i}, h(u_i))\}$, $\{d(h(u_i), t_{k_i+1})\}$ are bounded away from 0.

Let $\{[a_i, b_i]\}$ be a sequence of arcs in $J(E)$ having the following properties:

i) $[a_i, b_i] \cap V(E_1) = v_{k_i}$

ii) $\lim \{a_i\} = \lim \{b_i\} = w > u = \lim \{v_{k_i}\}$. This is possible since E_1 is separated, and each v_{k_i} is a strict local minimum of $J(E)$.

iii) Moreover $d(a_i, b_i)$ can be chosen small enough so that $h[a_i, b_i] \cap V(E_2) = \phi$, for all $i \in N$. This is possible since h is a homeomorphism and $h(u_i) \in (t_{k_i}, t_{k_i+1})$, where $\{d(h(u_i), t_{k_i})\}$ and $\{d(h(u_i), t_{k_i+1})\}$ are both bounded away from 0.

It follows from (ii) above that $\lim \{h(a_i)\} = \lim \{h(b_i)\} = h(w)$. Since $[h(a_i), h(b_i)]$ is a straight line segment for each $i \in N$, we conclude that $\lim \{[h(a_i), h(b_i)]\} = h(w)$. This is a contradiction since $\lim \{[a_i, b_i]\}$ is a nondegenerate line segment in E_1^* containing u and w.

Corollary 3.4. *Let* E_1 *and* E_2 *be separated* E*-continua and let* h: $E_1 \rightarrow E_2$ *be a homeomorphism. Then there exist skeletons* S_1 *and* S_2 *of* E_1 *and* E_2 *respectively and a homeomorphism* $g: \overline{S}_1 \rightarrow \overline{S}_2$ *satisfying the following conditions:*

i) $g|S_1$ *is order preserving;*

ii) $\{d(h(v),g(v)): v \in S_1\}$ *converges to* 0.

Proof. Since $V(E_1)$ is a V-sequence in E_1, it follows from Lemma 3.3 that $h(V(E_1))$ is a V-sequence in E_2. Let $g: V(E_1) \rightarrow V(E_2)$ be defined as follows: g(v) is the element of $V(E_2)$ which is closest to h(v), the distance being measured along $J(E_2)$. This function may be double valued for some points of $V(E_1)$, since h(v) may be the midpoint of a line segment whose endpoints are in $V(E_2)$. However, a repeated use of the fact that E_1 and E_2 are both separated and $h(V(E_1))$ is a V-sequence imply that g is actually a single valued bijection over some skeleton S_1 of E_1 onto a skeleton S_2 of E_2. Extend g on the rest of $\overline{S}_1$ as follows: $g|S_1^* = h|S_1^*$. Since $S_1^* = T(E_1)$, $S_2^* = T(E_2)$, it follows from Theorem 2.2 that $h(S_1^*) = S_2^*$. Hence g is now defined as a bijection from $\overline{S}_1$ onto $\overline{S}_2$. Condition (i) above is satisfied since $h|J(E_1)$ is order preserving. Condition (ii) follows from the construction of g and the fact that $h(S_1)$ is a V-sequence in E_2. Finally, the continuity of g follows from the continuity of h and the fact that $\{d(h(v),g(v)): v \in S_1\}$ converges to 0. This completes the proof.

Theorem 3.5. *Let* E *be a separated* E-*continuum with* T(E) = I *and let* h *be a self homeomorphism of* E *such that* $h|E^*$ *is order preserving. Then* $h|E^*$ *is the identity.*

Proof. By Corollary 3.4 we obtain skeletons S_1 and S_2 of E and a homeomorphism g: $\overline{S}_1 \to \overline{S}_2$ such that $g|S_1$ is order preserving and $g|S_1^* = h|S_1^*$. Since E is of type I, $(V(E))^* = S_1^* = S_2^* = E^* = I$. Hence it suffices to prove that $g|S_1^*$ is the identity. Since S_1 and S_2 are both skeletons of E, three cases arise:

i) $S_1 = S_2$. Then $g|S_1$ is the identity since it is order preserving. Hence $g|S_1^*$ is the identity.

ii) $g(S_1) = S_2 \subseteq S_1$. Let $v_0, v_1, v_2, \cdots$, be an enumeration of the elements of S_1. Since S_2 is a skeleton of E contained in S_1, $S_2 = v_k, v_{k+1}, \cdots$, for some $k \in N$. Since g is order preserving we have $g(v_i) = v_{i+k}$ for all $i \geq 0$. For each $i \geq 0$, let $0(v_i)$ denote the sequence $\{g^n(v_i)\}_{n \in N}$ where g^n denotes the nth self composition of g. Since $g(v_i) = v_{i+k}$, it follows that $v_i \in 0\ (v_{i \bmod k})$ for all $i \geq 0$. Hence $S_1 = \cup_{i=0}^{k-1} 0(V_i)$ and $S_1^* = E^* = \cup_{i=0}^{k-1} (0(v_i))^*$.

Suppose that $g|J(E)$ is a homeomorphism different from the identity. Then there exists some $v \in\ v_0, \cdots, v_{k-1}$ and some $x \in 0(v)$ such that $g(x) \neq x$. Two cases arise:

a) $g(x) > x$. Let $\{v_{i_j}\}$ be a subsequence of $0(v)$ converging to x such $|i_{j+1} - i_j| \geq 2$. Then $g(v_{i_j}) = v_{i_j+1}$ and $\lim \{v_{i_j+1}\} = g(x) > x$. Hence we may assume without loss of generality that $\pi_2(v_{i_j+1}) > \pi_2(v_{i_j})$ for all $j \in N$. Let r_j be the greatest integer in the set $\{1,\cdots,(i_{j+1} - i_j - 1)\}$ such that $\pi_2(v_{i_j+r_j}) \geq \pi_2(v_{i_j+1})$. Then $\pi_2(v_{i_j+r_j+1}) \leq \pi_2(v_{i_j+1})$ for all $j \in N$. Note that $i_j + r_j + 1 \leq i_{j+1}$. Let A be a convergent subsequence of $\{v_{i_j+r_j}\}$. Then $\lim A = y \geq \lim \{v_{i_j+1}\} = g(x) > x$. Since $g(A) \subseteq \{v_{i_j+r_j+1}\}$, it follows that $\lim g(A) = g(y) \leq g(x)$. Hence $y > x$ while $g(y) \leq g(x)$. This contradicts the assumption that $g|E^*$ is order preserving and hence increasing.

b) $g(x) < x$. Let $\{v_{i_j}\}$ be a sequence in $0(v)$ converging to x such that $i_{j+1} - i_j \geq 2$. Since $g(x) < x$, we may assume without loss of generality that $\pi_2(v_{i_j+1}) < \pi_2(v_{i_j})$ for all $j \in N$. Let r_j be the greatest integer in the set $\{1,\cdots,i_{j+1} - i_j - 1\}$. Then $\pi_2(v_{i_j+r_j+1}) \geq \pi_2(v_{i_j+1})$ for all $j \in N$. Let A be a convergent subsequence of $\{v_{i_j+r_j}\}$. Then $\lim A = y \leq g(x) < x$. Since $g(A) \subseteq \{v_{i_j+r_j+1}\}$, it follows that $\lim g(A) = g(y) \geq g(x)$. Hence $y < x$ and $g(y) \geq g(x)$. This again contradicts the assumption that $g|E^*$ is increasing and proves that $g|E^*$ is the identity.

iii) $g(S_1) = S_2 \supset S_1$. Then $g^{-1}(S_2) = S_1$. Replacing g by g^{-1} in case (ii) above and interchanging S_1 and S_2 we conclude that $g^{-1}|E^*$ is the identity and hence that $g|E^*$ is the identity. This completes the proof of the theorem.

Corollary 3.6. *If* h *is a self homeomorphism of a separated* E-*continuum* E *of type* I, *then* $h^2|E^*$ *is the identity.*

Proof. Note that $h^2|E^*$ is always order preserving and hence Theorem 3.5 applies.

The following theorem which is a direct consequence of Theorem 3.5, gives the desired sufficient conditions for FR.

Theorem 3.7. *Let* E *be an* E-*continuum satisfying the following conditions:*

i) E *is separated*

ii) T(E) = I

iii) E *admits only self homeomorphism which are order preserving on* E*. *Then* E ∈ FR.

4. Homeomorphism Classes in FR

Theorem 4.1. FR *contians uncountably many topologically distinct* E-*continua.*

Proof. The proof is constructive. Let A be a closed subset of $[o,\frac{1}{2}]$. We associate with A an E-continuum E_A by defining $V(E_A)$ as follows:

$V(E_A) = \{(a_i,b_i)\}$ where $\{a_i\}$ is a sequence in J converging to 0 and $\{b_i\} = p_1\ 0\ 1\ q_1\ 1\ 0\ 1\ 0\ p_2\ 0\ 1\ 0\ 1\ 0\ 1\ q_2\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ p_3 \cdots$ where (i) $\{p_i\}$ is an enumeration of the rationals in $[\frac{1}{2},1]$, (ii) $\{q_{2i+1}\}$ is an enumeration of the rationals in $[0,\frac{1}{2}]$, (iii) $\{q_{2i}\}$ is an enumeration of a dense subset A' of A in which each rational in A is repeated infinitely often. We prove that E_A satisfies the conditions of Theorem 3.7 and hence is an element of FR.

i) E_A is separated since $d((a_i,b_i),(a_{i+1},b_{i+1}) \geq \frac{1}{2}$ for each $i \in N$.

ii) E_A is of type I, since lim sup $\{p_i\} = [\frac{1}{2},1]$ and lim sup $\{q_i\} = [o,\frac{1}{2}]$.

iii) Every homeomorphism of E_A preserves the order of E_A^*. This will follow from the following more general result.

iv) Let A,B be two subsets of $[o,\frac{1}{2}]$ and let E_A, E_B be the associated E-continua described above. If $h: E_A \to E_B$ is a homeomorphism, then $h|E_A^*$ is order preserving and $h(A) = B$: Let $V(E_B) = \{(c_i,d_i)\}$, where $\{d_i\} = p_1\ 0\ 1\ q_1\ 1\ 0\ 1\ 0\ p_2\ 0\ 1\ 0\ 1\ 0\ 1\ q_2'\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ p_3 \cdots$.

By Corollary 3.4, there exists skeletons S_1 and S_2 of E_A and E_B respectively and a homeomorphism $g: \overline{S}_1 \to \overline{S}_2$ such that $h|S_1$ is order preserving. Let $S_1 = \{(a_i,b_i)\}_{i \geq i_o}$ and $S_2 = \{(c_i,d_i)\}_{i \geq i_1}$. Then $g(a_i,b_i) = (c_{i+k},d_{i+k})$, where $k = i_1 - i_o$. Suppose that $k \neq 0$, and let $b_{i_j} = q_{2j}$ for all $j \in N$. Then $g(a_{i_j},b_{i_j}) = (c_{i_j+k},d_{i_j+k})$. Since the q's occur with identical vertices in $\{b_i\}$ and $\{d_i\}$, it follows that d_{i_j+k} is an element of $\{d_i\}$ which is k terms away from q_{2j}'. However, it follows from the construction of $\{d_i\}$ that for all $j > \frac{|k|}{4} + 1$, $d_{i_j+k} \in \{o,1\}$. This implies that lim sup $\{g(a_{i_j},b_{i_j})\} \subseteq \{o,1\}$ whereas lim sup $\{(a_{i_j},b_{i_j})\}$ = lim sup $\{q_{2j}\} = [o,\frac{1}{2}]$. This is a contradiction. Hence $k = 0 = i_1 - i_o$ and $g(a_i,b_i) = (c_i,d_i)$ for all $i \geq i_o$. Since the 0's of the sequences $\{b_i\}$ and $\{d_i\}$ occur with identical indices, it follows that $g(a,o) = (c,o)$ for all $(a,o) \in V(E_A)$. Hence $g|E_A^*$ maps 0 to itself. Since $g|E_A^* = h|E_A^*$, it follows that

$h|E_A^*$ maps 0 to itself and hence is order preserving. Similarly, since the q_{2j}'s and the q'_{2j}'s occur with identical indices, it follows that $\pi_2(g(a_{i_j},q_{2j})) = q'_{2j}$. Hence $g(\limsup\{(a_{i_j},q_{2j})\}) = g(A) = \limsup\{(g(a_{i_j},q_{2j}))\} = \limsup\{(c_{i_j},q'_{2j})\} = \limsup q'_{2j} = B$. But $g(A) = h(A)$, since $h|E_A^* = g|E_A^*$. Hence $h(A) = B$.

The uncountability of the number of homeomorphism classes in FR now follows from Lemma 4.2.

Lemma 4.2. *There exists an uncountable collection of closed subsets of* $[o,\frac{1}{2}]$ *no two of which are order homeomorphic.*

Proof. The proof involves first associating with each sequence $P = \{p_i\}$ of ones and twos a closed subset A_P of $[o,\frac{1}{2}]$ as follows:

Let $b_1 > b_2 > b_3 > \cdots$ be a decreasing sequence of numbers in the open interval $(o,\frac{1}{2})$ converging to 0. Then $A_P = \{o\} \cup S \cup (\cup_{i=1}^{\infty}[b_{2i-1},b_{2i}])$, where S is a sequence of points in $(o,\frac{1}{2})$ converging to 0 and satisfying the following two conditions:

i) $S \subset \cup_{i=1}^{\infty}[b_{2i},b_{2i+1}]$

ii) $S \cap [b_{2i},b_{2i+1}]$ consists of exactly p_i points.

Let P and Q be two different sequences of ones and twos. Let A_P and A_Q be the associated closed subsets of $[o,\frac{1}{2}]$. Then A_P and A_Q are not order homeomorphic, since such a homeomorphism would map each interval $[b_{2i-1},b_{2i}]$ onto itself and hence the number p_i of points separating $[b_{2i-1},b_{2i}]$ and

$[b_{2i+1}, b_{2i+2}]$ in A_P must equal the number q_i of points separating $[b_{2i-1}, b_{2i}]$ and $[b_{2i+1}, b_{2i+2}]$ in A_Q. This contradicts the assumption that p_i is different from q_i for some $i \in N$. The result now follows since there are uncountably many different sequences consisting of ones and twos.

5. Necessary Conditions for FR

In Corollary 3.4, we proved that a homeomorphism between separated E-continua induces a homeomorphism between a pair of their skeletons. This was then used (Theorem 3.7) to give sufficient conditions for FR. In this section (Theorem 5.1) we prove a converse of Corollary 3.4. In fact the converse we prove is a little more general than the Corollary itself since 5.1 does not require E-continua to be separated. Theorem 5.1 is then used (Corollary 5.2) to give necessary conditions for FR.

Theorem 5.1. *Let* E_1 *and* E_2 *be any two* E*-continua, not necessarily separated, and let* S_1, S_2 *be skeletons of* E_1, E_2 *respectively. If* $h: \overline{S}_1 \to \overline{S}_2$ *is a homeomorphism preserving the order of* S_1, *then there exists a homeomorphism* $H: E_1 \to E_2$ *such that* $H|\overline{S}_1$ *and* h *agree except on at most the first point of* S_1.

Note that the condition that $h|S_1$ be order preserving is necessary since there are nonhomeomorphic E-continua which do have skeletons whose closures are homeomoprhic [3, p. 174].

Proof of Theorem 5.1. Let $v_1, v_2, v_3, \cdots; u_1, u_2, u_3, \cdots,$ be enumerations of the elements of S_1 and S_2 respectively.

Then $h(v_i) = u_i$ for all $i \in N$. Define $H\colon E_1 \to E_2$ as follows:

i) $H|E_1^*$ is any homeomorphism extending $h|S_1^*$. This is possible since S_1^* is closed in E_1^* and $E_2^* = [o,1]$.

ii) $H|[v,v_2]$ is an order preserving homeomorphism onto $[u,u_2]$ where v, u are the first elements of $J(E_1)$, $J(E_2)$ respectively.

iii) $H(v_i) = h(v_i)$ for all $i \geq 2$.

iv) Finally, if $t \in (v_i,v_{i+1})$ define $H(t) = \alpha H(v_i) + (1-\alpha)H(v_{i+1})$ where α is the unique number in $(o,1)$ such that $H(\pi_2(t)) = \alpha H(\pi_2(v_i)) + (1-\alpha)H(\pi_2(v_{i+1}))$.

It is obvious that H is bijective. Moreover, $H|J(E_1)$ is a homeomorphism onto $J(E_2)$; $H|E_1^*$ is a homeomorphism onto E_2^* and $H|\overline{S}_1$ is a homeomorphism onto $\overline{S}_2$. It remains to check the continuity of H at points of E_1^* as they are approached by sequences in $J(E_1)\setminus S_1$. Let $\{t_i\}$ be a sequence in $J(E_1)\setminus S_1$ converging to $t \in E_1^*$. Then $t_i \in (v_{k_i},v_{k_i+1})$, where $v_{k_i},v_{k_i+1} \in V(E_1)$. Due to compactness of E_1, we only need to consider cases where $\{v_{k_i}\}$ and $\{v_{k_i+1}\}$ are both convergent sequences. Suppose that $\lim \{v_{k_i}\} = p$ and $\lim \{v_{k_i+1}\} = q$. Then $p \neq q$, since E_1 is separated. Since $\{t_i\}$ converges to t, $\{\pi_2(t_i)\}$ converges to t and since $H|E_1^*$ is continuous, $\{H(\pi_2(t_i))\}$ converges to $H(t)$. For each $i \in N$, let α_i be the number in $(o,1)$ such that $H(\pi_2(t_i)) = \alpha_i H(\pi_2(v_{k_i})) + (1-\alpha_i)H(\pi_2(v_{k_i+1}))$. Since $H|E_1^*$ is continuous $\{H(\pi_2(v_{k_i}))\}$ converges to $H(p)$ and $\{H(\pi_2(v_{k_i+1}))\}$ converges to $H(q)$. Since $\{H(\pi_2(t_i))\}$ converges and

$H(p) \neq H(q)$, it follows that $\{\alpha_i\}$ converges to some number $\alpha \in [o,1]$. Hence

$$H(t) = \lim\{H(\pi_2(t_i))\} = \lim\{\alpha_i H(\pi_2(v_{k_i})) + (1 - \alpha_i)H(\pi_2(v_{k_i+1}))\}$$
$$= \alpha H(p) + (1 - \alpha)H(q).$$

It follows from part (iv) of the above construction of H, that $H(t_i) = \alpha_i H(v_{k_i}) + (1 - \alpha_i)H(v_{k_i+1})$. Hence $\lim \{H(t_i)\} = \alpha H(p) + (1 - \alpha)H(q) = H(t)$. This establishes the continuity of H on E_1^* and completes the proof of the theorem.

Corollary 5.2. *If* $E \in FR$, *then* E *is of type* I.

Proof. Let E be an E-continuum whose type is different from I. We will construct a function $H: E \to E$ such that $H|E^*$ is not the identity. Let S be any skeleton of E, and let $h: \overline{S} \to \overline{S}$ be the identity. By Theorem 5.1, there exists a homeomorphism $H: E \to E$ such that $H|\overline{S}$ and h differ on at most one point. Note that in part (i) of the construction of H in the proof of Theorem 5.1, $H|E^*$ was allowed to be any homeomorphism extending $h|S^*$. Since $S^* = T(E) \neq I$, $H|E^*$ can be chosen different from the identity. The rest now follows from Theorem 5.1.

6. Discussion

In Corollary 5.2 we proved that condition (ii) of Theorem 3.7 is necessary. Corollary 3.6 may lead one to question whether condition (iii) of Theorem 3.7 is redundant, since a self homeomorphism h of a separated E-continua E of type I, must satisfy the strong condition that $h^2|E^*$ is the identity. However, the author constructed separated

E-continua of type I that do admit self homeomorphism which reverse the order of the remainder. A general procedure for constructing such E-continua will be included in a subsequent paper. However, we believe that condition (i) of Theorem 3.7 is not necessary. In fact we propose the following:

Conjecture. An E-continuum $E \in FR$ iff E is of type I and every self homeomorphism of E is order preserving on the remainder.

The first half of this conjecture is already established in Corollary 5.2, and the second half can be obtained if one obtains an analogue of Corollary 3.4 for nonseparated E-continua.

References

[1] D. P. Bellamy, private communication to the author, August 1985.

[2] S. B. Nadler, Jr., *Continua whose cone and hyperspace are homeomorphic*, Trans. Amer. Math. Soc. 230 (1977).

[3] ________, *Hyperspaces of sets*, Marcel Dekker, Inc., New York, 1978.

[4] M. A. Awartani, *Compactification of the ray with the closed arc as remainder*, Top. Proc. 9 (2) (1984), 201-215.

[5] ________, *On almost continuous functions*, Proc. of the Texas Topology Symposium (1983), 221-228.

[6] H. Cook, *Continua which admit only the identity onto nondegenerate subcontinua*, Fund. Math. 60 (1967), 241-249.

Birzeit University

West Bank, Via Israel