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by

John W. Carlson

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Web: http://topology.auburn.edu/tp/
Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA
E-mail: topolog@auburn.edu
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Introduction

It is well known that on a set every filter is the intersection of the family of ultrafilters that contain it. The corresponding result fails in a topological space. That is; for example, not every open filter is the intersection of all the open ultrafilters that contain it. Those that are will be called balanced open filters. Balanced closed filters will be defined similarly.

Balanced closed filters have been used, Carlson [3], to characterize the closed subsets of the Wallman compactification of a T_1 topological space. A similar concept for balanced z-filters was used by Carlson [4] to characterize the closed subsets in the Stone-Čech compactification of a completely regular topological space. The one-to-one correspondence between the nonempty closed subsets of the hyperabsolute of a T_1 space and the balanced open grills is shown in Carlson [5]. Balanced collections of open ultrafilters have been used by Porter and Votaw, in [10] and [11], in their study of H-closed extensions. Our work will be related to theirs in section five of this paper.

In a natural way, the concepts of a balanced collection of minimal prime open filters and a balanced collection of minimal prime closed filters is introduced. Intersections of such collections will be called minimal balanced open or closed filters, as appropriate. An open filter $\mathcal{O}$ will be called closed generated provided there exists a closed filter $\mathcal{F}$ such that $\mathcal{O}$ is the open envelope of $\mathcal{F}$. Open generated closed filters will be defined similarly. It is shown that the open generated closed filters (closed generated open filters) are precisely the minimal balanced open filters (minimal balanced closed filters).

1. Preliminaries

Let (X,t) denote a T_1 topological space. Let $\mathcal{C}$ denote the collection of all the closed subsets in X.

Definition 1.1. Let $a \subset \mathcal{P}(X)$. Set:

(1) $\mathcal{F}(a) = \{F\colon F$ is closed and $X - F \notin a\}$

(2) $\mathcal{O}(a) = \{O\colon O$ is open and $x - O \notin a\}$

(3) $\mathcal{G}(a) = \{F\colon F$ is closed and these exist $A \in a$ with $F \supset A\}$

(4) $\mathcal{S}(a) = \{O\colon O$ is open and there exists $A \in a$ with $O \supset A\}$

(5) Sec $(a,t) = \{O \in t\colon O \cap A \neq \emptyset$ for each $A \in a\}$

(6) Sec $(a,\mathcal{C}) = \{F \in c\colon F \cap A \neq \emptyset$ for each $A \in a\}$

$\mathcal{G}(a)$ is called the closed envelope of a and $\mathcal{S}(a)$ is called the open envelope of a. If a is a collection of open sets then Sec (a,t) will be denoted by Sec (a); if a is a collection of closed subsets then Sec $(a,\mathcal{C})$ will be denoted by Sec (a). In general; Sec^2 $(a,\mathcal{B})$ = Sec (Sec $(a,\mathcal{B})$, $\mathcal{B}$).

Definition 1.2. A prime open filter P is a nonempty collection of open sets satisfying

(1) $\emptyset \notin P$

(2) O open and $Q \in P \Rightarrow O \in P$

(3) $O \in P$ and $Q \in P \Rightarrow O \cap P \in P$

(4) O and Q open and $O \cup Q \in P \Rightarrow O \in P$ or $Q \in P$

A prime closed filter is defined similarly.

Lemma 1.3. *Let U be a nonempty collection of open sets and V a nonempty collection of closed sets.*

(1) Sec $(U,C) \subset F(U)$

(2) Sec $(V,t) \subset O(V)$

(3) Sec $(U,t) = O(G(U))$

(4) Sec $(V,c) = F(S(V))$

Let K be a prime closed filter and P a prime open filter. Set $K^* =$ Sec (K,t) and $P^* =$ Sec (P,C). Salbany, in [11], has shown that K^* is a prime open filter and P^* is a prime closed filter. By Lemma 1.3, $P^* = F(P)$ and $K^* = O(K)$.

Definition 1.4. Let O be an open filter. Set $b(O) = \cap\{M: M$ is an open ultrafilter and $M \supset O\}$.

An open filter O is said to be balanced provided $O = b(O)$; that is, O is equal to the intersection of all of the open ultrafilters that contain it.

Definition 1.5. Let F be a closed filter Set $b(F) = \cap\{N: N$ is a closed ultrafilter and $N \supset F\}$. A closed filter F is said to be balanced provided $F = b(F)$; that is, F is equal to the intersection of the family of closed ultrafilters that contain

(1) A closed filter F is said to be open generated if there exists an open filter O such that $F = G(O)$.

(2) An open filter O is said to be closed generated if there exists a closed filter F such that $O = S(F)$.

Definition 1.6. A minimal prime open filter is prime open filter that is minimal in the collection of prime open filters.

Minimal prime closed filters are defined similarly.

Theorem 1.7. *Let O_1 and O_2 be nonempty collections of open sets and F_1 and F_2 be nonempty collections of closed sets.*

(1) $O_1 \subset O_2$ *implies* $G(O_1) \subset G(O_2)$

(2) $O_1 \subset O_2$ *implies* $F(O_2) \subset F(O_1)$

(3) $F_1 \subset F_2$ *implies* $S(F_1) \subset S(F_2)$

(4) $F_1 \subset F_2$ *implies* $O(F_2) \subset O(F_1)$

Theorem 1.8. *Let K be a nonempty collection of closed sets and P a nonempty collection of open sets.*

(1) *K is a prime closed filter iffi $O(K)$ is an open prime filter.*

(2) *K is a minimal prime closed filter iffi $O(K)$ is an open ultrafilter.*

(3) *K is a closed ultrafilter iffi $O(K)$ is a minimal prime open filter.*

(4) *P is a prime open filter iffi* $\mathcal{F}(P)$ *is a prime closed filter.*

(5) *P is a minimal prime open filter iffi* $\mathcal{F}(P)$ *is a closed ultrafilter.*

(6) *P is an open ultrafilter iffi* $\mathcal{F}(P)$ *is a minimal prime closed filter.*

(7) $K = \mathcal{F}(\mathcal{O}(K))$

(8) $P = \mathcal{O}(\mathcal{F}(P))$

The following result is used repeatedly in the sequel.

Theorem 1.9. *Let M be an open ultrafilter and N a closed ultrafilter. Then:*

(1) $S(N) = \mathcal{O}(N)$

(2) $G(M) = \mathcal{F}(M)$

The mapping S preserves convergence and the mapping G preserves adherence in the sense made precise by the following theorem.

Theorem 1.10. *Let* $\mathcal{F}$ *be a closed filter and* $\mathcal{O}$ *be an open filter.*

(1) $x \in \text{adh } \mathcal{F}$ *implies* $x \in \text{adh } S(\mathcal{F})$

(2) $G(\mathcal{O}) \to x$ *implies* $\mathcal{O} \to x$

(3) $x \in \text{adh } \mathcal{O}$ *iffi* $x \in \text{adh } G(\mathcal{O})$

(4) $\mathcal{F} \to x$ *iffi* $S(\mathcal{F}) \to x$.

It is easy to see that $\mathcal{O} \to x$ does not imply that $G(\mathcal{O}) \to x$ nor does $x \in \text{adh } S(\mathcal{F})$ imply that $x \in \text{adh } \mathcal{F}$.

2. Prime open filters and prime closed filters

On a set X, a prime filter is an ultrafilter. Moreover, each filter on a set is the intersection of the family of ultrafilters that contain it and each grill is precisely the union of a nonempty family of ultrafilters.

In a topological space a prime open filter need not be an open ultrafilter nor is a prime closed filter necessarily a closed ultrafilter. Moreover; open filters are not necessarily the intersection of the family of open ultrafilters that contain them. Indeed, these are the balanced open filters. Similarly, an open grill is not necessarily the union of family open ultrafilters and the corresponding statements are true for closed filters and closed grills.

However, it can be shown [5], that the prime open filters generate the open filters and the open grills in the same way as ultrafilters generate filters and grills. Similarly for prime closed filters. This is made precise in the following theorem.

Theorem 2.1.

(1) *Every open filter equals the intersection of the family of prime open filters that contain it.*

(2) *Every closed filter equals the intersection of the family of prime closed filters that contain it.*

(3) *Every open grill is precisely the union of a family of prime open filters.*

(4) *Every closed grill is precisely the union of a family of prime closed filters.*

Notation 2.2. Let $\mathcal{F}$ be a closed filter and $\mathcal{O}$ an open filter. Then there exist index sets Λ and Ω such that

$$\mathcal{F} = \cap\{\mathcal{K}\alpha: \alpha \in \Lambda\}$$

$$\mathcal{O} = \cap\{\mathcal{P}\alpha: \alpha \in \Omega\}$$

where $\mathcal{K}\alpha$ is a prime closed filter for each $\alpha \in \Lambda$ and $\mathcal{P}\alpha$ is a prime open filter for each $\alpha \in \Omega$.

Theorem 2.3. *Let* X *be a* T_1 *topological space. The following statements are equivalent.* (Frolik [6])

(1) *Every prime open filter is an open ultrafilter.*

(2) *Every prime closed filter is a closed ultrafilter.*

(3) *Each closed ultrafilter* $\mathcal{F}$ *satisfies the following equivalent properties:*

(A) $F \in \mathcal{F}$ *implies there exists* $G \in \mathcal{F}$ *and* $\mathcal{O}$ *open with* $G \subset \mathcal{O} \subset F$

(B) $F \in \mathcal{F}$ *implies that* int $F \in \mathcal{S}(\mathcal{F})$

(C) $\mathcal{F} = \mathcal{G}(\mathcal{S}(\mathcal{F}))$.

(4) X *has the discrete topology.*

The mappings $\mathcal{O}$ and $\mathcal{F}$ interchange convergence and adherence in the sense made precise by the following theorem.

Theorem 2.4. *Let* $\mathcal{P}$ *be a prime open filter and* $\mathcal{K}$ *a prime closed filter. Then:*

(1) $\mathcal{F}(\mathcal{P}) \to x$ *implies* $x \in \text{adh } \mathcal{P}$

(2) $\mathcal{K} \to x$ *implies* $x \in \text{adh } \mathcal{O}(\mathcal{K})$

(3) $x \in \text{adh } \mathcal{K}$ *iffi* $\mathcal{O}(\mathcal{K}) \to x$

(4) $\mathcal{P} \to x$ *iffi* $x \in \text{adh } \mathcal{F}(\mathcal{P})$.

It is easy to see that $x \in \text{adh } \mathcal{P}$ does not imply that $\mathcal{F}(\mathcal{P})$ converges to x nor does $x \in \text{adh } \mathcal{O}(\mathcal{K})$ imply that $\mathcal{K} \to x$.

3. Balanced open filters and balanced closed filters

As defined in section 1, an open filter is called balanced provided it is equal to the intersections of the family of open ultrafilters that contain it. A balanced closed filter is defined similarly.

Lemma 3.1. (Carlson [3]). *Let* $\mathcal{O}$ *be an open filter and* $\mathcal{F}$ *a closed filter on a* T_1 *topological space* X.

(1) $\text{Sec }(\mathcal{O}) = \cup\{\mathcal{M}: \mathcal{M}$ an open ultrafilter and $\mathcal{O} \subset \mathcal{M}\}$

(2) $\text{Sec}^2\ (\mathcal{O}) = \cap\{\mathcal{M}: \mathcal{M}$ an open ultrafilter and $\mathcal{O} \subset \mathcal{M}\}$

(3) $\text{Sec }(\mathcal{F}) = \cup\{\mathcal{N}: \mathcal{N}$ a closed ultrafilter and $\mathcal{F} \subset \mathcal{N}\}$

(4) $\text{Sec}^2\ (\mathcal{F}) = \cap\{\mathcal{N}: \mathcal{N}$ a closed ultrafilter and $\mathcal{F} \subset \mathcal{N}\}$

Theorem 3.2. (Porter and Votaw [11]). *Let* $\mathcal{F}$ *be an open filter on* X *and* $\mathcal{G} = \cap\{\mathcal{U}: \mathcal{U}$ is an open ultrafilter with $\mathcal{F} \subset \mathcal{U}\}$. *Then* $\mathcal{G} = \{U: U$ is open and $\text{int } \overline{U} \in \mathcal{F}\} = \mathcal{F} \vee \mathcal{D}$ *where* $\mathcal{D} = \{U: U$ is open and dense$\}$.

Easily $\mathcal{G}$ in the above theorem by Porter and Votaw corresponds to our $b(\mathcal{F})$.

Intuitively, an open filter $\mathcal{O}$ is not balanced if there exists an open set that is "almost" in $\mathcal{O}$ in the sense that the open set contains no members of $\mathcal{O}$ but its closure does. It follows that a balanced open filter must contain each open dense set.

Theorem 3.3. *Let* X *be a* T_1 *topological space and set* $S = \cap\{M: M$ an open ultrafilter on X$\}$. *Let* O *be an open filter on* X. *Then:*

(1) S *is the smallest balanced open filter on* X

(2) $S = \{O: O$ is an open dense set$\}$

(3) $b(O) = \{Q \in t:$ there exists $O \in O$ with $O \subset \overline{Q}\}$

(4) $b(O)$ *is the smallest balanced open filter containing* O.

(5) (Porter and Votaw [11]) $b(O) = O \vee S$

Theorem 3.4. *Let* O *be an open filter. The following statements are equivalent.*

(1) O *is balanced and prime*

(2) $O \cup Q \in \mathrm{Sec}^2(O)$ *implies* $O \in O$ *or* $Q \in O$.

Theorem 3.5. *Let* F *be a closed filter. The following statements are equivalent.*

(1) F *is a balanced closed filter.*

(2) *For each closed set* $G \notin F$ *there exists an open set* $O \supset G$ *such that* $F \not\subset O$ *for each* $F \in F$.

(3) *If* F *is a closed set and each open* O *that contains* F *belongs to* $S(F)$ *then* $F \in F$.

Intuitively, a closed filter F is not balanced if there exists a closed set F that is "almost" in F in the sense that every open set that contains F also contains a member of F.

Theorem 3.6. *Let* X *be a* T_1 *topological space and* F *a closed filter on* X.

(1) $b(\mathcal{F}) = \mathcal{F} \cup \{G$: G is closed and for each $O \in t$ with $O \supset G$ there exists $F \in \mathcal{F}$ with $F \subset O\}$.

(2) *If* X *is normal then each balanced prime closed filter is a closed ultrafilter.*

Theorem 3.7. *Let* $\mathcal{F}$ *be a closed filter and* $\mathcal{O}$ *an open filter. Then:*

(1) $S(\mathcal{F}) = S(b(\mathcal{F}))$

(2) $G(\mathcal{O}) = G(b(\mathcal{O}))$

4. Open generated closed filter and closed generated open filters

By Theorem 3.7, we have that the open envelope of a closed filter equals the open envelope of the smallest balanced closed filter that contains the filter; that is, $S(\mathcal{F}) = S(b\mathcal{F})$ for each closed filter $\mathcal{F}$. The corresponding result holds for open filters.

Theorem 4.1. *A closed filter* $\mathcal{F}$ *is open generated if and only if* $cl(int\ F) \in \mathcal{F}$ *for each* $F \in \mathcal{F}$.

Theorem 4.2. *Let* $\mathcal{F}$ *be a closed filter and* $\mathcal{O}$ *an open filter.*

(1) *If* $\mathcal{F}$ *is prime then* $S(\mathcal{F}) \subset \mathcal{O}(\mathcal{F})$ *and* $S(\mathcal{F}) = \mathcal{O}(\mathcal{F})$ *iffi* $\mathcal{F}$ *is a closed ultrafilter.*

(2) *If* $\mathcal{O}$ *is prime then* $G(\mathcal{O}) \subset \mathcal{F}(\mathcal{O})$ *and* $G(\mathcal{O}) = \mathcal{F}(\mathcal{O})$ *iffi* $\mathcal{O}$ *is an open ultrafilter.*

(3) *If* $S(\mathcal{F})$ *is an open ultrafilter then* $\mathcal{F}$ *is a closed ultrafilter.*

(4) *If* $G(O)$ *is a closed ultrafilter then* O *is an open ultrafilter.*

(5) *If* X *is normal then* $S(F) = S(G(S(F)))$.

(6) *If* X *is normal then* O *is closed generated iffi* $O = S(G(O))$.

Theorem 4.3. *Let* F_1 *and* F_2 *be closed filters and* O_1 *and* O_2 *be open filters.*

(1) $S(F_1) = S(F_2)$ *iffi* $b(F_1) = b(F_2)$.

(2) $G(O_1) = G(O_2)$ *iffi* $b(O_1) = b(O_2)$.

Definition 4.4. Let S be a specified type of filter. A nonempty collection C will be called a balanced collection of type S provided:

(1) if $F \in S$ and $F \supset \cap C$ then $F \in C$, and

(2) if $F \in S$ and $F \subset \cup C$ then $F \in C$.

(For our work, S will be the: open ultrafilters, closed ultrafilters, minimal prime open filters, or minimal prime closed filters.) An open filter O is called a minimal balanced open filter provided there exists a balanced collection P of minimal prime open filters such that $O = \cap P$. Minimal balanced closed filters are defined similarly.

Every balanced open (closed) filter is the intersection of a balanced collection of open (closed) ultrafilters. Also, if $\{N_\alpha: \alpha \in \Omega\}$ is a balanced collection of closed ultrafilters then $S(\cap\{N_\alpha: \alpha \in \Omega\}) = \cap\{S(N_\alpha): \alpha \in \Omega\}$.

Theorem 4.5. *Let* X *be a topological space. Let* F *be a closed filter and* O *an open filter. Then:*

(1) *F is an open generated closed filter if and only if F is a minimal balanced closed filter.*

(2) *O is a closed generated open filter if and only if O is a minimal balanced open filter.*

Proof. Let F be an open generated closed filter. Then there exists an open filter O_1 such that $F = G(O_1)$. Let $O = b(O_1)$. Then, O is a balanced open filter and by Theorem 3.6, $F = G(O_1) = G(b(O_1)) = G(O)$. Since O is balanced, the family $\{M: O \subset M, M \text{ an open ultrafilter}\}$ is a balanced collection of open ultrafilters. Let Ω be an index set for this collection. Then $O = \cap\{M_\alpha: \alpha \in \Omega\}$ and $F = G(O) = \cap\{G(M_\alpha): \alpha \in \Omega\} = \cap\{F(M_\alpha): \alpha \in \Omega\}$. Now $\{F(M_\alpha): \alpha \in \Omega\}$ is a balanced collection of minimal prime closed filters and thus F is a minimal balanced closed filter.

If F is a minimal balanced closed filter there exists a balanced collection of minimal prime closed filters $\{K_\alpha: \alpha \in \Omega\}$ such that $F = \cap\{K_\alpha: \alpha \in \Omega\}$. Now $\{O(K_\alpha): \alpha \in \Omega\}$ is a balanced collection of open ultrafilters and $O = \cap\{O(K_\alpha): \alpha \in \Omega\}$ is a balanced open filter. Now, $G(O) = \cap\{G(O(K_\alpha)): \alpha \in \Omega\} = \cap\{F(O(K_\alpha)): \alpha \in \Omega\} = \cap\{K_\alpha: \alpha \in \Omega\} = F$. Thus, F is an open generated closed filter. The proof of (2) follows in a similar manner.

5. Applications

The concept of a balanced collection of open ultrafilters was called "saturated" by Porter and Votaw [11] in their study of H-closed extensions of Hausdorff spaces.

In this section, O^y denotes the trace filter of $y \in Y$ when Y is an extension of X and σX denotes the Fomin H-closed extension of X.

Theorem 5.1. (Porter and Votaw [11]). *Suppose there exists a continuous function from* σX *onto an* H-*closed extension* Y *of* X *that leaves* X *pointwise fixed. Let* $F \subset Y \setminus X$. *The following are equivalent:*

(a) F *is closed in* Y

(b) F *is compact*

(c) $\cap\{O^y: y \in F\}$ *is a free open filter and if* $\cap\{O^y: y \in F\}$ *meets* O^z *for some* $z \in Y \setminus X$ *then* $z \in F$.

If in the above theorem we let $Y = \sigma X$ and the identity map on Y be the continuous map we have that condition (c) is equivalent to saying that $\{O^y: y \in F\}$ is balanced collection of open ultrafilters; or equivalently, that $\cap\{O^y: y \in F\}$ is a balanced free open filter. Thus, we have the following theorem.

Theorem 5.2. *Let* X *be a Hausdorff topological space. Then there exists a* 1-1 *correspondence between the balanced free open filters on* X *and the nonempty closed subsets of* $\sigma X - X$, *the remainder of the* Fomin H-*closed extension of* X.

Essentially, Porter and Votaw [11], in this interesting paper, partition $\sigma X - X$ into closed subsets, replace each closed subset by the corresponding balanced open filter and use the quotient topology to obtain a simple H-closed extension of X. Simple here means that the new extension has the simple extension topology in the sense of Banaschewski [1].

The following sequence of results lead to a relationship between the Fomin H-closed extension and the Stone-Čech compaction of a space.

Definition 5.3. An open filter $\mathcal{O}$ is called regular provided $O \in \mathcal{O}$ implies there exists $Q \in \mathcal{O}$ with $\overline{Q} \subset O$. (Equivalently, $\mathcal{O} = S(G(\mathcal{O}))$.)

Lemma 5.4. *Let $\mathcal{M}$ be an open ultrafilter. $\mathcal{M}$ is regular iffi $\mathcal{M}$ is closed generated.*

Theorem 5.5. *Let* X *be a regular space. The following statements are equivalent.*

(A) *Every free open ultrafilter is regular.*

(B) *Every free prime open filter is a free open ultrafilter.*

Proof. (A) => (B). Suppose $\mathcal{P}$ is a free prime open filter that is not an open ultrafilter. Then there exists an open ultrafilter $\mathcal{M}$ such that $\mathcal{P} \subset \mathcal{M}$ and $\mathcal{P} \neq \mathcal{M}$. Now, by A and Lemma 5.4 there exists a closed ultrafilter $\mathcal{N}$ such that $\mathcal{M} = \mathcal{O}(\mathcal{N})$. Hence $F(\mathcal{P}) \supsetneq F(\mathcal{M}) = F(\mathcal{O}(\mathcal{N}))$, which is impossible. Thus, $\mathcal{P}$ is a free open ultrafilter.

(B) => (A). Let $\mathcal{M}$ be a free open ultrafilter. Set $\mathcal{K} = F(\mathcal{M})$. Then $\mathcal{K}$ is a minimal prime closed filter and there exists a closed ultrafilter $\mathcal{N} \supset \mathcal{K}$. Let $\mathcal{P} = \mathcal{O}(\mathcal{N})$. Then, $\mathcal{P} = \mathcal{O}(\mathcal{N}) \subset \mathcal{O}(\mathcal{K}) = \mathcal{O}(F(\mathcal{M})) = \mathcal{M}$. By condition (B), $S(\mathcal{N}) = \mathcal{O}(\mathcal{N}) = \mathcal{P} = \mathcal{M}$ provided $\mathcal{P}$ is free.

$\mathcal{P}$ is clearly a minimal prime open filter. In order to show that $\mathcal{P}$ is free, we first note that adh $\mathcal{K} = \cap\mathcal{K} \subset \{\overline{M}: M \in \mathcal{M}\} = \emptyset$. Similarly, $\mathcal{K} \subset \mathcal{N}$ implies $\mathcal{N}$ is a free closed ultrafilter. Now $\mathcal{P} = \mathcal{O}(\mathcal{N}) = S(\mathcal{N})$. Let $x \in \cap\overline{S(\mathcal{N})}$. Now $\{x\} \notin \mathcal{N}$ and so there exist $N \in \mathcal{N}$ with $x \notin N$. Since X is regular there exists disjoint open sets O_x and O_N containing x and N, respectively. Now $O_N \in S(\mathcal{N})$ and $x \notin \overline{O}_N$. Thus, $\mathcal{P}$ is a free open filter.

Hence, we have that $\mathcal{M} = S(\mathcal{N})$, where $\mathcal{N}$ is a free closed ultrafilter. $\mathcal{M}$ is regular by Lemma 5.4.

Theorem 5.6. (Porter and Votaw [10]). *Let* X *be Hausdorff.* σX *is the* Stone-Čech *compactification of* X *iffi* X *is regular and every free open ultrafilter is regular.*

The following theorem follows from Theorems 5.5 and 5.6.

Theorem 5.7. *Let* X *be Hausdorff.* σX *is the* Stone-Čech *compactification iffi* X *is regular and every free prime open filter is a free open ultrafilter.*

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University of South Dakota

Vermillion, South Dakota 57069