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NOTES ON THE HISTORY OF GENERAL TOPOLOGY IN RUSSIA*

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This Memoir is dedicated to the Memory of my friend Ben Fitzpatrick who wanted this article to be written

My notes are definitely very incomplete. It is an approximation but not a dense approximation of the real picture. None of the topics is covered really in a systematic way. Some are almost left aside, like dimension theory where a very good survey [114] is available. I left out topics involving applications of homological methods and homotopy methods to general spaces, in particular, the theory of shape. The works of Alexandroff, Chogoshvili, Kuz'minov, Sklyarenko, Zarelua, Shtan'ko, Bogatyi going in that direction are not discussed. Thus, this is only a sketch, and it gives only a part of the picture, but it could hardly be different: to present a reasonably detailed complete picture would require a book of at least 200 pages. However, the surveys [5], [7], [8], [21], [29], [114], [27], [93] provide a lot of additional material relevant to the subject and well complement this article.

General Topology in Russia began with Pavel Sergeevich Alexandroff, a student of N.N. Luzin. He entered Moscow University in the autumn 1913, 17 years old. In his own words, he already knew everything that the students in mathematics were taught in their first year. So, not to waste time, he was spending much time in the mathematical library of the University. With delight, he found there Cantor's memoirs on set theory. As Alexandroff wrote in [7]: "... I began to read Cantor in the original and learned what transfinite numbers are, a new world opened before me, ... and I was in a state of excitement." Here, in this excitement, we now see one of the sources of Topology in Russia. Alexandroff attended D.F. Egorov's lectures and seminars, and became a student of N.N. Luzin, who was deeply interested in the descriptive theory of sets. In the spring 1915, Luzin suggested to Alexandroff, then an undergraduate student in mathematics at Moscow University, to prove that the cardinality of every uncountable Borelian subset of the real line is precisely 2^ω. Alexandroff solved the problem in summer 1915, and to do so, he introduced the notion of A-operation (the name was given by Souslin).

In fact, Alexandroff proved that if a Borel subset A of a complete separable metrizable space is uncountable, then it contains a topological copy of the Cantor set. (Independently, this was also proved by F. Hausdorff).

He presented his result in a lecture at the student mathematical society of Moscow University on October 13, 1915. Among students who came to his talk were P.S. Urysohn and M.Ya. Souslin, he met them there for the first time, and we can only guess how the beauty of Alexandroff's result and the force of his presentation influenced these two brilliant young fellows.

Alexandroff's paper with the theorem was published in 1916 [1].

So these are the events and dates that mark the very beginnings of General Topology in Russia.

Looking at the developments in General Topology in Russia during the years 1916–1940, we distinguish the following major directions of research:

1. Completeness and descriptive set-theory.

2. Compactness and local compactness.

3. Metrization and generalized metrics.

4. Dimension theory.

5. Behaviour of topological invariants under continuous mappings satisfying certain additional restrictions.

Here are some major achievements in these directions during the first years of topology in Russia.

In 1916 Souslin proved that not every A-set (that is, a set obtained from closed sets by Alexandroff's A-operation) is Borelian.

In 1920 Souslin posed his famous problem, which made such a great impact on the developments of set-theoretic methods [147].

In 1924 Alexandroff proved that a separable metrizable space is metrizable by a complete metric if and only if it is homemorphic to a G_δ-subset of a compact metrizable space [2]. The concept of a locally finite covering was also introduced by Alexandroff in 1924 (see [8]). It was established, in modern terminology, that every separable metrizable space is paracompact. The concept of paracompactness was defined much later, in 1944, by Dieudonné, and in 1948 the famous theorem of A.H Stone was proved.

We should mention here the next theorem of M.A. Lavrent'ev (1924), who was also a student of Luzin. Suppose X and Y are complete metric spaces, A is a subspace of X, B is a subspace of Y, and h is a homeomorphism between A and B. Then h can be extended to a homeomorphism between some G_δ-subsets of X and Y containing, respectively, A and B [86].

During 1921 and 1922 Alexandroff and Urysohn, who became close friends, did research in depth on the notion of compactness. Many results were obtained by them by February 1923. However, the article was published only much later, after Urysohn's death, in 1929, in the now famous Memoir on compact spaces [13]. This was first systematic treatment of compactness, and how modern it is still now! Definitions were given in the most natural form and at that level of abstraction which not only brings the required level of generality to theorems, but also makes their proofs most transparent. In this paper we enjoy a perfect balance between beautiful theorems and fascinating examples. The lexicographically ordered square, the double circumference, the double arrow space are full of meaning and grace, we always keep them close to us.

One of the important results in the Memoir is the following theorem, which strongly influenced, by its formulation and its proof, the development of the theory of cardinal invariants of topological spaces, an important and rich direction of research nowadays: the cardinality of any perfectly normal compact space does not exceed 2^ω.

In the proof of this theorem the idea of an uncountable ramified family of sets played a vital role.

The notion of a perfectly normal compactum later turned out to be closely related to Souslin Problem. It was while working on the Memoir in 1921–1922, and in connection with the above theorem, that Alexandroff and Urysohn formulated their conjecture that the cardinality of every first countable compact Hausdorff space does not exceed 2^ω. The work on this conjecture contributed much to the growth of the theory of cardinal invariants and to the developments of set-theoretic techniques in the study of compactness.

In the Memoire, the notion of an H-closed space (a Hausdorff space which is closed in every larger Hausdorff space) was introduced. The authors proved that every regular H-closed space is compact, and gave an intrinsic criterium for a Hausdorff space to be H-closed. This concept became a subject of systematic investigation much later, and many interesting results were obtained (by M.H. Stone, S.V. Fomin, M.G. Katetov, J. Porter, and others).

Memoire also contains the important and nontrivial characterization of compactness in terms of points of complete accumulation. In the proof of it, for the first time topology came in direct contact with the delicate questions of the theory of cardinal numbers.

Here are some other results obtained in Russia in twenties:

Luzin and Souslin proved that every Borelian subspace of a Polish space is a one-to-one continuous image of a closed subspace of the space of irrational numbers.

Alexandroff and Urysohn provided a nice topological characterization of the space of irrational numbers [12].

Urysohn proved his famous lemma [161], characterizing normality in terms of continuous functions, and established the metrizability criterium for the separable case: a space X is homeomorphic to a separable metrizable space if and only if X is a normal T_1-space with a countable base [162]. Urysohn was also the first to construct a countable normal T_1-space with a single nonisolated point which is not first countable (and therefore, is not metrizable). For that, he used rapidly growing sequences of natural numbers, whose complements were taken as neighbourhoods of the ideal point. See also [160].

Alexandroff and Urysohn also produced a general metrization theorem [11], built upon the idea of development and a version of the concept of star refinement. Their proof is based on Chittenden's metrization theorem, formulated in terms of restrictions on generalized metrics.

In 1921 Menger and Urysohn independently defined the small inductive dimension *ind* and proved some fundamental theorems about it, in particular, the following:

If $indX = n$ for a separable metric space X, then X is the union of $n+1$ (and not less) of its zero-dimensional subspaces.

Urysohn also proved that for each compact metric space X all the three (by now, classical) dimensions coincide: $dimX = indX = IndX$ [159], [163]. This fundamental identity was extended in 1925 by W. Hurewicz and L.A. Tumarkin to all separable metric spaces [155], [156], which, of course, was a gigantic step forward, which strongly influenced further developmment of dimension theory.

It took more than 30 years to clarify what remains from these identities in larger classes of spaces, such as the class of all metrizable spaces and the class of all compact Hausdorff spaces (by Katetov, Morita, Filippov, Fedorchuk, Pasynkov, Lifanov, and others).

One of main achievements of Urysohn in dimension theory was his theory of Cantor manifolds described in his "Mémoire sur les multiplicités Cantorienne" (see more about it in [8]). This theory influenced Alexandroff in his work on general homological dimension theory.

In one of his last papers, Urysohn defined and studied a homogenous universal separable metric space U. It is homogeneous, in a natural sense, with respect to any two isometric finite subsets of U, and it is unique with respect to the properties mentioned above [164].

In a recent paper (1998) A.M Vershik brings forward an important connection between Urysohn's universal metric space U and the Gromov-Hausdorff distance between metric spaces X and Y. This distance can be described as the infimum of Hausdorff distances between images of X and Y in U under all possible isometric embeddings of X and Y in U.

Some relatively recent interesting applications of Urysohn's universal space belong to V.V. Uspensky.

In 1926 Alexandroff introduced the concept of a nerve of a family of subsets (see [8]). This is, probably, one of the most important notions introduced by Alexandroff. The whole theory of approximation of spaces by polyhedra is based upon it, it is a key step in defining spectral representations of spaces.

Here is a relevant quotation from Alexandroff and Fedorchuk's paper [8]:

"The concept of nerve, applied to directed systems of covers (of one kind or another) and the concept of projective spectrum based on it provided a means of reducing properties of topological spaces, first of all, of compact metrizable spaces, to those of complexes and their simplicial mappings. This made it possible, above all, to define and study homological invariants, in the first place, of compact metric spaces, then of all compact spaces (Alexandroff) and of more general spaces (Čech) on the basis of the concept of nerve."

The twenties also witnessed the beginnings of the theory of compactifications of topological spaces in the works of A.N. Tychonoff.

Tychonoff introduced the notion of the product of arbitrary family of topological spaces (called now *the Cartesian*, or *Tychonoff* product), and proved one of the most important theorems of set-theoretic topology: that the product of any family of compact spaces is compact [152]. This theorem opened the door to a wealth of natural new examples of compact spaces, such as the Tychonoff and Cantor cubes of arbitrary weight.

On this basis, Tychonoff characterized subspaces of compact Hausdorff spaces as completely regular T_1-spaces (1930) (now we call them *Tychonoff spaces*) [152]. The method of diagonal products of mappings, introduced by Tychonoff in the proof of his embedding theorem, later became one of standard methods for constructing embeddings.

Tychonoff was also the first to consider the compactum $\beta\omega \setminus \omega$ as an example of an infinite compact space without isolated points the topology of which cannot be described in terms of convergent sequences, since there are no such nontrivial sequences in the space at all [153].

Tychonoff's definition of the product topology was based on his clear understanding that a topology cannot be always described in terms of convergent sequences; so he did not try to define the product topology by specifying which sequences converge.

Later Tychonoff moved to differential equations and to computer science. He was the founder of the Computer Science faculty at Moscow University. However, he always highly valued his background in pure mathematics, in general topology, in particular. When Alexandroff was seventy, Tychonoff found in his archive the notes of the topology course he has taken from Alexandroff in

1922/23, edited those with my help, printed out the manuscript and presented it to Alexandroff at the solemn meeting in his honour.

In 1935 Tychonoff proved an important general theorem on fixed points [154], generalizing similar results obtained earlier for Banach spaces. He established that if X is a nonempty convex compact subspace of a locally convex linear topological space then, for every continuous mapping of X into itself, there exists a fixed point. This is one of the "linear-topological" principles which have many applications in mathematics and play a unifying role.

In 1940 S.V. Fomin developed a theory of H-closed extensions of topological spaces, generalizing the theory of compactifications to the Hausdorff case. One of his first good results was an elementary and transparent proof of M.H. Stone's theorem (obtained in 1935) that if every closed subspace of a Hausdorff space X is H-closed, then X is compact [62].

One of Alexandroff's colleagues at Moscow University at the end of twenties was V.V. Niemytzkii. Like Alexandroff himself, he came to topology from Luzin's famous seminar on functions of real variable, which was flourishing between 1917 and 1928. Niemytzkii did pioneering work on what is now called the theory of generalized metric spaces. This theory stems from the natural idea that, since not all spaces are metrizable, one should look for some weaker restrictions on the distance structure which would make larger classes of spaces "metrizable" in the new sense. In particular, Niemytzkii defined Δ-metrics as the "metrics" that do not have to satisfy the symmetry condition [101]. He also introduced symmetrics, as "metrics" that need not satisfy the triangle inequality.

Niemytzkii proved that every symmetrizable first countable compact Hausdorff space is metrizable, and established the similar statement for Δ-metrizable compacta [102]. In a joint paper, Alexandroff and Niemytzkii [9] characterized developable spaces as those the topology of which is generated by a symmetric satisfying a certain natural Cauchy condition.

One of Alexandroff's students, A.S. Parhomenko, was among the first who worked on Alexandroff's question (related to a problem of Banach): when a space can be condensed (that is, mapped by a continuous one-to-one mapping) onto a compact Hausdorff space. He proved that if we throw away countably many points from a metrizable compactum, what remains can be condensed onto a (metrizable) compactum [106], [107]. As we now know, a similar statement is not true in the class of first countable compact Hausdorff spaces (Pytkeev E. G.). On the other hand, in 1975 V. I. Belugin proved that the complement to a countable subset in a zerodimensional first countable compact space can be always condensed onto a compact Hausdorff space [32].

Parhomenko also introduced and characterized minimal Hausdorff spaces.

In 1935 an International Conference in Topology was organized in Moscow. Alexandroff and Kolmogorov were most instrumental in it. This was a big event, many outstanding mathematicians came. This helped to strengthen the natural ties among topologists from all over the world.

In thirties and forties a significant progress was made in the dimension theory. L.S. Pontryagin, who became Alexandroff's student in 1928, proved, in a joint paper with Tolstowa [123], that every separable metric space X such that $dimX \leq n$ is embeddable in Euclidean $(2n + 1)$-space R^{2n+1} (1931). Independently, the same result was obtained by Lefschetz and Nöbeling.

A.N. Kolmogorov presented the first example of a dimension-raising open continuous mapping with zero-dimensional preimages defined on a compact metric space [75]. It was already known from one of the earlier works of Alexandroff that every nonempty metrizable compactum can be represented as a continuous image of of the Cantor set. Later this line of investigation was very successfully extended by L.V. Keldysh [73].

Alexandroff's student, N.B. Vedenisov (killed later in the war) proved that $dimX \leq IndX$ for each normal space X [167].

Alexandroff himself proved that $dimX \leq indX$ for every compact Hausdorf space [AL40].

In 1939 Pontryagin's book "Topological groups" was published (in Princeton, in English) [122]. Besides such achievements as duality theory for locally compact Abelian groups and theorems on algebraic structure of such groups, it contained some important purely topological constructions of a general nature, such as the definition of the Σ-product of a family of spaces, and the proof of the theorem that every topological group satisfying the T_1-separation axiom is Tychonoff.

Note, that it is still unknown whether every regular paratopological group is Tychonoff.

Another famous result, obtained in 1939, is the Gel'fand-Kolmogoroff theorem stating that whenever X is a compact Hausdorff space, the ring $C(X)$ of continuous real-valued functions on X determines the space X upto a homeomorphism [64]. This paper was one of the starting points for Gelfand's theory of commutative normed rings. A central role in this theory belongs to the natural connection between such rings and certain compact spaces elements of which are maximal ideals of the ring (see [63], [65]). This is one of the main gates through which nowadays general topology enters the realm of functional analysis.

J. S. Očan, another student of N.N. Luzin, who worked mostly in classic descriptive set theory, introduced in 1940 topologies on spaces of subsets, which bear his name but sometimes are also called Pixley-Roy topologies (see [103]).

It was in 1935, two years before A. Weil introduced the fundamental notion of a uniform space, that V.A. Efremovich proposed the concept of a proximity space. However, his paper remained unpublished until 1951 [48], [49].

The general theory of proximity spaces, based on the axioms introduced by Efremovich, was developed at the beginning of fifties by Yu.M. Smirnov [144]. He also established the connection between proximity spaces and uniform spaces. One of the main results of Smirnov on proximity spaces was the theorem on the natural one-to-one correspondence between proximities compatible with a given Tychonoff topology on X and all Hausdorff compactifications of the space X.

Later, in the fifties, V. M. Ivanova and A.A. Ivanov generalized the notion of a proximity space and created a corresponding theory of "adjacency or contiguity spaces" (see [70] and [69]). This also provided an approach to the theory of compactifications. The theory of contiguity spaces was later developed by H. Herrlich into a more general theory of nearness spaces.

In [145] Yu. M. Smirnov developed dimension theory of proximity spaces.

M.F. Bockshtein, whose main interests were lying in algebraic topology, proved in 1948 his lemma on separating disjoint open subsets in the product of uncountably many separable metrizable spaces by a projection onto some countable subproduct [33]. This contributed to a better understanding of the topology of uncountable Tychonoff and Cantor cubes.

Major developments in general topology at the end of forties and in the beginning of fifties are connected with the notions of metrizability and paracompactness.

Paracompactness was introduced by J. Dieudonné in 1944. After A.H. Stone proved in 1948 his celebrated theorem on paracompactness of all metrizable spaces, the path to general metrizability criteria was open, and, almost simultaneously, three topologists, R.H. Bing, Jun-iti-Nagata, and Yu.M. Smirnov proved the following theorem: a regular T_1-space X is metrizable if and only if it has a σ-locally finite base [146] (Bing's formulation was slightly different). The technique, by which A.H. Stone proved his theorem, lead to the notion of a fully normal space based on the concept of star refinement, playing very important role in the theory of uniform spaces as well.

At this time, in the middle of fifties, a new, postwar, generation filled University auditoriums. One of those who returned from the war was I.A. Vainstein. In 1947 he proved his famous lemma which was so important for understanding of closed mappings [165]. He proved that for every closed continuous mapping of one metric space into another the boundaries of preimages of points are automatically compact. From this he derived the conclusion that, for every closed continuous mapping f of a metric space X onto a metric space Y, there exists a closed subspace Z of X such that $f(Z) = Y$ and the restriction of f to Z is a perfect mapping [166]. This allows to reduce closed mappings to perfect mappings in the class of metrizable spaces. Vainstein's lemma was analyzed, generalized and applied in the years to follow uncountably many times.

In 1949 A. G. Lunz and O. V. Lokutzievskij made a next important step in dimension theory of nonmetrizable spaces, a step which considerably influenced further developments in the theory. A compact Hausdorff space X such that $dimX = 1$ and $indX = IndX = 2$ was constructed (see [89]). It was also a counterexample to addivity of Ind under sums of finite collections of closed subspaces.

For the personal carrier of P.S. Alexandroff the beginning of the fifties was also an important time: he was elected a full memeber of the Academy of Science of USSR (1954, he was 58). This gave him an additional influence and responsibility, and impetus, and, probably, was one of the reasons why he decided to organize a new seminar on topology for beginners, for undergraduate students of the second and the third year of studies (basically; of course, some older student also would come and were welcome). I should mention, that in the academic year 54/55, my first year, Alexandroff was our lecturer in Analytic Geometry, so we knew him and many of us were very impressed with him and his appearance.

This seminar became a new generic point of growth of set-theoretic topology in Russia.

The Seminar met twice in May 1955, and started its regular work in September 1955. It would meet weekly, for two academic hours. The first sessions of the seminar were attended by more than thirty students. Gradually, the number diminished, and in the middle of the year about ten were left. Almost everyone of these was very enthusiastic about his work in the seminar. Among the students were myself, Ponomarev, Pasynkov, Ivanovskij, Chernavskij (all second year). Alexandroff was helped in his work with students by V.A. Uspensky, than a graduate student of A.N. Kolmogorov, working in Mathematical Logic, A.S. Parhomenko, a faculty member at the time, and O.V. Lokutsievskij, a former student of Alexandroff, an expert in dimension theory, who was simultaneously working in applications.

Alexandroff had not been doing set-theoretic topology for some time in that period; so we all had to start from the very beginning. By that time, on my own initiative, I had already read R. Baire's book on discontinuous functions and on transfinite numbers and was very impressed. I also proved myself that rational numbers are not G_δ, so I was prepared for this seminar and was a step ahead, which was not bad. The first topics in the seminar were the properties of subsets of the real line, points of condensation, Cantor set, compactness in metric spaces, completeness, the Baire theorem, and so on. We were reading some articles on the suggestion of Alexandroff and then presented talks about the main results in these at the seminar. So at first we were mostly refereeing, we were welcome to simplify the proofs or to present our own proofs of the theorems refereed.

Very soon we moved to reading original articles. I remember reading Smirnov's systematic articles on proximity spaces and compactifications, and, of course, the Memoire on compact Hausdorff spaces. We studied articles on metrization. In particular, I was impressed by the wealth of material and of new notions in Bing's famous article on metrization in which he proved metrizability results for Moore spaces, introduced the fundamental concept of collectionwise normality, and constructed far reaching examples. Of course, we all studied the proof of A.H. Stone's theorem on paracompactness of metrizable spaces. The main problem at that time was to find time for other mathematical disciplines: we still were undergraduate students with three more years to go in that capacity!

Approximately at this time Michael's classic papers on paracompactness criteria started to appear, and they were immediately refereed in our seminar.

In the second year of the work of the seminar its leadership was strengthened: L.V. Keldysh joined it. Several students worked under her guidance in these years; A.V. Chernavskij and M.A. Shtan'ko among them. L.V. Keldysh represented the geometric direction in the set-theoretic topology, though her earlier papers were on descriptive set theory. She had been a student of Luzin, a member of the famous Luzitania.

My first result in topology was obtained in the Spring of 1957: I characterized Čech-complete spaces by a restriction on a sequence of open coverings of the space. Alexandroff was not particularly impressed, so it remained unpublished until Frolik came to Moscow in 1960 or 1961 and presented a manuscript to Alexandroff with a similar characterization of Čech completeness. Then Alexandroff presented simultaneously his article to Doklady and mine to Vestnik of Moscow University (see [17]). I also showed in the article that locally Čech complete spaces need not be Čech complete.

My first published paper appeared in Doklady AN SSSR in 1959 [16]. I introduced in it the notion of a network in a space, and the class of spaces with a countable network, and applied the notion of network to give a short and natural proof of the theorem: if a compact Hausdorff space X is the union of two subspaces of weight not greater than an infinite cardinal number τ, then the weight of X also does not exceed τ. This was based on the theorem that the weight of a compact Hausdorff space is equal to its network weight. This theorem also implies that under a continuous mapping of a space onto a compact Hausdorff space the weight cannot increase. In particular, it was established in [AR59] that every compact Hausdorff space which is a continuous image of a separable metrizable space, is also metrizable. These were my first results on cardinal invariants; they were obtained in my Master's Thesis.

Ponomarev's first paper appeared in 1957; he was studying in detail some properties of multivalued mappings, the topic which only gained in importance with years. Special importance to this subject was added by the celebrated theorem of E. Michael on the existence of continuous selections of a multivalued lower semicontinuous mapping of a zero-dimensional paracompact space in a complete metric space.

One of the first Ponomarev's results, proved a little later, is the theorem that every first countable space can be represented as the image of a metrizable space under an open continuous mapping [118].

In 1958, using spectra and powerful techniques from the theory of compact groups, L. Ivanovskij (then an undergraduate student of the same year as the three of us) solved a major problem posed by Alexandroff [71]: he proved that every compact topological group is a dyadic compactum. After that Ivanovskij moved to algebraic topology and became a student of M. M. Postnikov; this was his only result in set-theoretic topology, so far as I know. The same result was obtained independently by V.I. Kuzminov [84], also a partcipant of our seminar and a student of P.S. Alexandroff.

In 1960 I noticed that $dimG = indG$ for every locally compact topological group (the fact that every locally compact group is automatically strongly paracompact played the key role in the proof). B.A. Pasynkov strongly improved this result by showing that, under the same restrictions, also $dimG = IndG$ [111]. To prove this, he used and developed further the method of inverse spectra; this was his first research topic in topology.

The problems of dimension theory were one of the central topics in Alexandroff seminar. It was also the subject of common interest for Alexandroff, Smirnov, on one side, and L.V. Keldysh on the other side, and the same was true for their students.

The work in this direction was greatly influenced by L.V. Keldysh's example [73] of an open continuous mapping with zero-dimensional fibers which maps a one-dimensional compact metric space onto the square I^2. On the basis of this result, Pasynkov later obtained a general theorem in the same direction (see [10]). In connection with his theorem Pasynkov introduced interesting technique of partial products which, in particular, stimulated the development of the concept of resolution of a space (see [53], [54]).

In 1957 L.V. Keldysh published a paper in which it was shown that every Euclidean cube of dimension greater than 3 can be represented as an image of the three dimensional cube under an open continuous mapping with connected fibers [74] (such mappings are called monotone). This was a solution of Alexandroff's problem and a great achivement in geometric topology.

The classic theorem of Hurewicz for dimension-lowering mappings of separable metrizable spaces was extended by E.G. Sklyarenko to paracompact spaces [142]. Sklyarenko also expanded Pasynkov's theorem on the equivalence of the three dimensions to quotients of locally compact topological groups [143]. Sklyarenko graduated one year earlier than us; he was a student of Smirnov, who joined Alexandroff as one of co-leaders of the seminar. Main interests of Smirnov at that time were in dimension theory, in theory of compactifications, in proximity spaces, and in uniform spaces. Though we were not his direct students, his research and his interests influenced us as well, and the theme of compactifications became recurrent in the seminar and in our research.

One of first results of Sklyarenko was the following theorem [141]: every normal space has a compactification preserving both the dimension dim and weight. The proof of it is based on Mardešić's, by now classic, factorization theorem for *dim* for continuous mappings of compact spaces (1960). Pasynkov and Zarelua applied Mardešić's factorization theorem to prove (independently, in 1964) that there exists a universal compactum (with regard to embeddings) for compact Hausdorff spaces of given weight and fixed dimension [112], [172].

One may conclude from the history of the early years of our seminar that our success depended not only on our enthusiasm and individual effort, and not only on support and interest of Alexandroff, but on the good knowledge of the most fresh results obtained by other mathematicians, on deep interest to work of other topologists, such as A.H. Stone, E. Michael, K. Morita, M. Katetov, and others.

An important event in the life of some young members of Alexandroff seminar happened in August 1961. We (Arhangel'skii, Pasynkov, Ponomarev, Sklyarenko, and some older topologists) went abroad, with Alexandroff, to the first Prague Topological Symposium. That was the first international conference in our life. We met outstanding mathematicians whom we knew only by their articles. M.H. Stone, E. Hewitt, M.G. Katetov, J. Isbell, E. Michael, E. Moise, S. Eilenberg, D. Kurepa, meeting each of them was an event in my life. I think, Michael, Hewitt, Eilenberg, Isbell, among others, were at my talk, which was devoted to networks in compacta and to the addition theorem for weight in compacta. On the spur of the moment I made the talk in English, though I did not know it at that time very well. Later, in the corridor, I was blamed for that by a topologist from Poland and a topologist from Yugoslavia, they told me that I should have talked in Russian, not in English.

Our encounter with E. Michael in Prague became a starting point of a long, and very important for me, correspondence. In 1965 Michael introduced additional restrictions on the notion of network, about which I talked in Prague, and defined $\aleph_0$-spaces. In particular, all quotients of separable metrizable spaces are $\aleph_0$-spaces, while all continuous images of separable metrizable spaces have a countable network. Michael also renamed spaces with countable network into cosmic spaces.

I remember a conversation with J. Isbell in Prague in 1961. He asked me if I knew the proof of A.S. Mischenko's [97] theorem that every point countable base of a compact Hausdorff space is countable. This theorem was recently obtained by a new, younger, member of Alexandroff's seminar, A.S. Mischenko, and was not yet published. I knew the proof and explained it to Isbell. He told me then that Mary Ellen Estill had obtained a particular case of this theorem (for perfectly normal compacta) by a very different argument. Later it became clear that Mary Ellen's argument could be easily transformed into a proof of the general case of the theorem. That is how and when I heard about Mary Ellen Rudin for the first time.

Though the interests of the participants of our Moscow seminar in topology were quite broad, in Prague really immense horizons opened before us. Lectures by E. Hewitt and M.H. Stone on applications of topology in functional analysis, talks on applications of topology in theory of numbers, talks on geometric topology, and so on: we saw a whole new world taking shape and colours before our eyes. I should also mention that from the end of seventies visitors from abroad very often came to our seminar and worked closely with some of its participants. Among them I remember Miroslav Katetov, J. Nagata, Richard Engelking, Kazimierz Kuratovskij, Wlodek Holsztynski, Andrjej Lelek, Rostislav Telgarski, Edwin Hewitt. Isztvan Juhasz also came for a rather short period of time, as a student, and then withdrew himself to Hungary. All these contacts influenced us enormously, they all helped us to see new perspectives in topology. Many more visits happened later.

We returned to Moscow from Prague inspired and happy, and full of memories of this beautiful old city, and impressed with hospitality of our Čech colleagues.

In October 1961 three of us, Pasynkov, Ponomarev, and myself, were accepted into the faculty of mechanico-mathematical faculty, we became members of Alexandroff's Chair "Of higher geometry and topology". I think, Sklyarenko became a member of the same chair the year before.

In his talk in Prague, Alexandroff directed the attention to the behaviour of topological invariants under continuous mapping satisfying various restrictions. By that time, many results of this kind had been obtained in dimension theory. Some five years earlier, by now a classic paper by M. Henriksen and J. Isbell on perfect mappings appeared. Several important properties were shown to be invariant under perfect mappings. Note, that the study of perfect mappings of general, not necessarily locally compact, spaces was initiated by I. A. Vainstein in 1947 [165].

After the Prague Symposium, I became especially interested in this topic. I tried not only to establish that a certain property is preserved or not preserved under a certain type of mappings, but to use restrictions on mappings to compare different classes of spaces, to establish a system of connections between classes of spaces, whose original definitions were given independently and did not involve mappings at all. Later this direction of research obtained the name "Mutual classification of spaces and mappings". Some results of this kind were already well known: Hurewicz's theorems in dimension theory, Morita's results obtained later, Ponomarev's characterization of first countable spaces as open continuous images of metrizable spaces. Approximately at this time Ponomarev was working on the theory of absolutes of topological spaces in which he established strict connection between the class of extremally disconnected paracompact spaces and the class of all paracompact spaces [119]. The notions of an absolute of a topological space and a perfect irreducible mapping played a central role in the theory. Categorical aspects of the theory of extremally disconnected spaces were earlier brought to the light by A. Gleason, who constructed the theory of absolutes in the class of compact Hausdorff spaces. In the years to follow S.D. Iliadis, V. I. Ponomarev, L.B. Shapiro, V.M. Ul'yanov have done much of interesting research on extremal disconnectedness and absolutes of spaces (see [67], [68], [121], [157]).

To make mutual classification of spaces and mappings possible, a sufficiently rich list of natural restrictions on mappings was required. The restrictions were imposed on preimages of points or on the behaviour of open and closed sets under mapings. The notions of a quotient mapping, of a pseudo-open (=hereditarily quotient) mapping, of monotone mapping, of a finite-to-one mapping and so on were systematically considered and applied in this context.

I noticed at this time that spaces with a uniform base, introduced by Alexandroff in [6], are precisely the images of metrizable spaces under open continuous mappings with compact fibers. Ponomarev observed that images of metrizable spaces under open continuous mappings with separable fibers are precisely the spaces with a point countable base. These results showed that mappings can indeed be used effectively for comparing different topological properties.

I myself became interested in a problem formulated by Alexandroff at the First Prague Symposium in Topology: to characterize preimages of metrizable spaces under perfect mappings. I did this in 1963; the key step was to introduce the notion of a p-space. I remember the special feeling I had at the moment when this notion crystallized in my mind; I was traveling at that time together with A. Chernavskij in a Moscow bus. A Tychonoff space X is a perfect preimage of a metrizable space if and only if X is a paracompact p-space [19], [20]. The class of p-spaces embraces the classes of all metrizable spaces and of all locally compact Hausdorff spaces. Actually, it includes also all Čech complete spaces (note, however, that a metrizable space is Čech complete if and only if it is metrizable by a complete metric; therefore, not all p-spaces are Čech complete). It turned out that the class of p-spaces is very interesting in many respects. All Tychonoff Moore spaces are p-spaces. All p-spaces with a countable network are separable and metizable. In the presence of the p-property paracompactness and the Lindelöf property become countably productive. Later p-spaces were applied in various situations, for example, in the study of joint continuity (A. Bouziad), in the theory of topological groups (Pasynkov studied almost metrizable groups (1964), which are, in fact, just topological groups which are p-spaces).

Since paracompact p-spaces can be described in terms of perfect mappings, and from this description it is clear that every perfect preimage of a paracompact p-space is a paracompact p-space, the next problem came naturally to the light: is any image of a paracompact p-space under a perfect mapping again a paracompact p-space?

The new results and open questions on the connections between spaces in terms of mappings accumulated rapidly, and in 1966, after defending my second, Doctor of Mathematical Science dissertation (one of my referees was A.N. Kolmogoroff, who liked the notion of p-space), I wrote my first survey article on the subject for Uspechi Mat. Nauk (Russian Math. Surveys). This was in summer 1966, I did not have much time to do that, so I wrote it in two weeks. The article, called "Mappings and spaces" (see [21]), helped to bring attention to the research done in this direction in Moscow. In particular, several American mathematicians became explicitly interested in the subject and contributed greatly to the study of p-spaces and to other topics in the theory of mappings and spaces, in particular, D. Burke and J. Worrell. In that article I also discussed various natural restrictions on bases of spaces. Alexandroff introduced the notion of a uniform base in 1960 [6]. Then, strengthening the restriction in Alexandroff's definition, I introduced the notion of a regular base, and proved that every T_1-space with a regular base is metrizable (1961). A regular base is, of course, a rather strong structure. However, the criterion has its advantages. First, we do not have to impose additional restrictions on separation in the space. Second, and the most important advantage, is that regular bases can be viewed as "carriers" of the paracompactness property of metrizable spaces. Indeed, given a regular base $\mathcal{B}$ in a space X, there is a simple "mechanism" for inscribing a locally finite family of open sets into any open covering γ of X: one just has to take first the family $\mathcal{B}_\gamma$ of all elements of $\mathcal{B}$ contained in at least one element of γ and then to take all maximal elements of the family $\mathcal{B}_\gamma$, this is the desired locally finite refinement of γ.

A brilliant achievement of J. Worrell was an example of a perfect mapping of a p-space X onto a Tychonoff space Y which is not a p-space. The manuscript still exists but, unfortunately, it was never published.

In 1966 I met for the first time H.H. Wicke; he came to the International Congress on Mathematics in Moscow. By that time H.H. Wicke and J. Worrell had done some very delicate and significant work on the spaces with bases of countable order, the notion I introduced in 1963 [18]. One of amazing properties of bases of countable order they established is that if a Hausdorff space has such a base locally, then it has it globally. A similar statement about σ-discrete or σ-locally finite bases (and many other bases) is not true. The result implies that no paracompactness type property follows from the existsence of a base of countable order. Besides, every Moore space has a base of countable order.

At least one problem from my 1966 survey remains open until now. This is the question whether every metacompact Tychonoff space can be represented as an image of a paracompact (Hausdorff) space under an open continuous mapping with compact fibers.

Sometime at the beginning of sixties, I proved the following theorem: every locally compact perfectly normal metacompact space is paracompact. Alexandroff thought that this result is equivalent to some statement in his and Urysohn's Memoire on compact spaces and I could not convince him that this is not the case. Because of that he did not recommend the paper for publication and it remained unpublished until much later (1971), when Frank Tall formulated it as an open question at the third Prague Symposium. Then I submitted the paper to Doklady [26], via Alexandroff, who finally recognized his mistake and was ready to correct it.

Also, around 1963, I formulated a natural problem in dimension theory which was partially solved only recently. I asked at the seminar and in several other audiences whether the three classic dimensions dim, ind, and Ind coincide for all Tychonoff spaces with a countable network. It was clear that, in this situation, $dimX \leq indX$ and $indX = IndX$, so it remained to show that $indX \leq dimX$. Much work on this problem was done later, especially, in Georgia, then a republic of USSR, where a group of students, working under direction of Lazo Zambahidze, developed a systematic approach to dimension type invariants (see [171]). However, a real progress on the problem was made only in nineties, when a counterexample was constructed under CH by George Delisthatis and Stephen Watson. It still remains unknown whether there is a similar counterexample in ZFC (I conjecture, it exists). It is also not clear what happens for $\aleph_0$-spaces or for Tychonoff quotients of separable metrizable spaces, whether all three dimensions coincide for such spaces. It is another old problem remaining open.

The second Prague Topological Symposium took place in 1966. Many participants of our Moscow seminar in General Topology again attended it (myself among them). This Symposium became the main place to present our results and to get new ideas and new inspiration for our work. For some reason I did not talk on my results in the classification of spaces and mappings (on p-spaces, in particular). This was done by P.Alexandroff in his major talk. I presented just my most fresh result: that if every subspace of a Hausdorff space X is a k-space, then X is Fréchet-Urysohn. I spent much time with Z. Frolik at the Symposium.

In 1965 V.V. Filippov became my student. He just came to me and said that he wants to solve some difficult problem in General Topology. I stated to him the problem of the invariance of the class of paracompact p-spaces under perfect mappings. I already tried to do this myself and felt this is a difficult question; I had formulated the question in the seminar and it remained open for several years. I suspect, it would have remained unsolved much longer if not for Filippov: the same year he came to me with the complete solution! That was his first published paper [56]. In his next paper Filippov expanded to paracompact p-spaces the theorem of A.S. Mischenko on compact Hausdorff spaces with a point countable base: every paracompact p-space with a point countable base is metrizable [57]. Filippov aslo proved a remarkable theorem on preservation of metrizability: if a compact Hausdorff space Y is an image of a metrizable space X under a quotient mapping with separable fibers, then Y is metrizable (1969) [58]. This greatly generalized Mischenko's theorem, since, according to a result of Ponomarev, every space with a point countable base is an image of a metrizable space under an open continuous mapping with separable fibers.

In 1964 another young fellow approached me asking to be his advisor in topology. His name is M.M. Choban. He was my first student in topology who came to me as an undergraduate student. Choban was sent to Moscow from Tiraspol Pedagogical Institute by P.K. Osmatesku (the famous organizer of Tiraspol' Symposia), who recently got his Ph.D. at Moscow University and went back to Moldova to teach there. There he discovered Mitrofan Choban, and being a kind person, immediately arranged for Mitrofan to be transferred to Moscow University. Mitrofan was given an explicit instruction the first thing to find me there. However, Mitrofan started his life in Moscow with some sports activities, soon broke his hand, and when he finally came to me, his hand was still in bandages.

It took about one year for Mitrofan to learn the basic things in general topology. Then he immediately embarked on his own adventures in this domain. He came to me often to discuss things, I was posing questions to him and was giving references. It was clear that he was already living in the fascinating world of set-theoretic topology.

The first results of Choban were on p-spaces [40], and on preservation of metrizability. In particular, he proved in 1966 that if a separable first countable regular space Y is an image of a metrizable space X under a quotient mapping with separable fibers, then Y is metrizable [38]. This was influenced by a result of A.H Stone, who established the theorem in the case when X is separable (1956).

Then Choban moved to study the space $\mathcal{Z}(X)$ of all compact subsets of a space X in the Vietoris topology [39], [42]. This space plays an important role in the theory of multivalued mappings and in the theory of selections, which was demonstrated by E. Michael in his classic paper (1951). Results of Ponomarev on multivalued mappings also influenced Choban. He obtained a series of new results on the exponential space, published in 1969–1971. In particular, Choban established that $\mathcal{Z}(X)$ is Čech complete if and only if X is Čech complete, and a similar statement for local compactness.

In later years Choban proved some strong generalizations of Michael's selection theorems. Of course, he and other general topologists were impressed with the celebrated theorem of M.I. Kadets stating that all separable Banach spaces are homeomorphic [72]. A selection theorem of Michael played an essential role in the proof.

Choban's interests also expanded to descriptive set theory; he was trying to extend it as much as possible beyond the limits of the classic case. In particular, it was not clear for some time, what part of descriptive set theory can be extended from the case of metrizable compacta to the case of perfectly normal compacta (see [138]).

By that time, A. V. Chernavskij had already proved the existence of an A-set which is not a Borel set in every uncountable perfectly normal compactum [34]. After that (in 1966) Ponomarev established, under the same restrictions on X, that every Borel class is nonempty [120]. Choban [41] (and, independently, Jayne (1970)) obtained the following definitive result: If X is a non-scattered compact Hausdorff space, then all Baire classes of functions on X are nonempty. Choban's results spread in all directions of descriptive set-theory and provide for this theory the most general setting.

We should mention here another result of Chernavskij, the proof of which required an excellent command of technique of set-theoretic topology and of algebraic toplogy. He established that for every finite-to-one open and closed continuous mapping of a connected manifold there exists a natural number k such that all preimages of points have cardinality not greater than k [35].

V.V. Filippov, after his first successes in theory of mappings, changed the direction of research (that was rather typical for participants of our seminar: quite a few participants of it kept moving from one domain of general topology to another). And again his motivation was to solve a most difficult problem. He asked me for advice and I recommended him to work on a famous question from dimension theory, the hardness of which was already tested by several participants of our seminar. I asked him to prove or to disprove that $indX = IndX$, for every compact Hausdorff space X

Filippov brilliantly solved this problem in 1969 in the negative [59]. His example was quite complicated, and in years to follow several papers (by Filippov himself, V.V. Fedorchuk, B.A. Pasynkov and I.K. Lifanov) were devoted to constructing simpler examples or to examples of compacta satisfying some additional properties. In particular, in 1970 Flippov constructed a compact Hausdorff space X such that $dimX = 1$, $indX = 2$, and $IndX = 3$, and in 1969 he constructed a first countable compact space X such that the dimensions ind and Ind are not equal for X (see [60], [61]).

While Filippov was my student, Fedorchuk was Alexandroff's student; he joined our seminar around the end of fifties. The first papers of Fedorchuk were devoted to linearly ordered spaces (1966–1969). Then he started to work in dimension theory. In 1971 he constructed a homogeneous compact Hausdorff space such that $indX \neq IndX$ [53]. Further description of the progress in Russia in dimension theory made in those years can be found in the survey by Alexandroff and Fedorchuk [8] (see also [114]).

Compactness was always at the center of research in Alexandroff seminar. And Alexandroff himself was always particularly interested in the class of dyadic compacta, which he introduced. Dyadic compacta are defined as Hausdorff continuous images of generalized Cantor's discontinua (Cantor's cubes) D^τ (where τ is any cardinal number and D is the two-points discrete space). Marczewski noted that the compactum $\omega_1 + 1$ is not dyadic, since the Souslin number of every dyadic compactum is countable. Thus, not every compact Hausdorff space is dyadic, in contrast with Alexandroff's theorem that every nonempty metrizable compactum is a continuous image of D^ω [3]. The interest in dyadic compacta considerably increased after Ivanovskij and Kuzminov proved (1958) that every compact topological group is a dyadic compactum [71], [84]. Once I was exposed to the importance of the theorem of Ivanovskij and Kuzminov in a somewhat unusual way. I was talking to a well known mathematician working not directly in topology but with strong interest in topological algebra, and he mentioned to me that he was strongly impressed at the end of fifties with my result that every compact group is a dyadic compactum. I answered that this was not my theorem. Then he asked: "Well, then, what have you done?" I must confess, I felt really ashamed that I did not prove that theorem.

From the very beginning it was clear that cardinal invariants must play an important role in the theory of dyadic compacta. First of all, as we already mentioned, the Souslin number of each dyadic compactum is countable. Even more is true: every uncountable family of nonempty open sets in a dyadic compactum X contains an uncountable subfamily with nonempty intersection (that is, $\aleph_1$ is a caliber of X). This was proved by N.A. Shanin in [127]. Shanin actually proved that every uncountable regular cardinal number is a caliber of any dyadic compactum. Shanin also proved that every dyadic compactum which is also a linearly ordered topological space is metrizable [128] (see also [129]).

Another beautiful result on dyadic compacta, involving cardinal invariants, was obtained in (1948) by a student of Alexandroff, and the son of a famous Russian poet, A.S. Esenin-Vol'pin. He established in 1949 that every first countable dyadic compactum is metrizable [52]. Later on A.S. Esenin-Vol'pin moved to mathematical logic and did valuable research there.

At the beginning of sixties, a student of V.I. Ponomarev, B.A. Efimov, started to work on dyadic compacta. In 1963 he proved that for a dyadic compactum to be metrizable it suffices that it be first countable at a dense set of points [43]. Efimov also established that every hereditarily normal dyadic compactum is metrizable, and that if a dyadic compactum is Frechet-Urysohn, then it is metrizable (1963). See also [47]. Later on, in 1968, in a joint paper of myself and Ponomarev [28], it was proved that every dyadic compactum of countable tightness is metrizable. Looking at these results now, we can say that the theory of cardinal invariants had as one of its sources the theory of dyadic compacta.

Clearly, the class of dyadic compacta is not closed hereditary. However, Efimov established [45] that every closed G_δ-subset of a dyadic compactum is dyadic. A particularly nice result of Efimov is his theorem that every compact Hausdorff space which is a continuous image of the Σ-product of a family of metrizable compacta is metrizable [44].

A very special dyadic compactum X was constructed by V. V. Pashenkov in [110]: the space X is zero-dimensional, homogeneous, but is not homeomorphic to any generalized Cantor cube D^τ. Moreover, it is not a Dugundji compactum. Pashenkov developed for that an original "extension" technique.

In 1967 my interests in topology extended to topological groups. Pontryagin's book "Continuous groups" [122] contained a very good introduction into the subject, it was written with much care, proofs were given with all details. All natural infinite groups come with some natural topology, and this topology need not be always discrete, so a topological group is a most natural, harmoniously balanced object. On the other hand, while the theory of linear topological spaces was already well developed, the general theory of topological groups, beyond the limits of the class of locally compact groups, was in its most elementary stage. I was also fascinated by the fact that, in the presence of a group structure "synchronous" with a topology, the relations between topological invariants change so drastically (for example, first countability becomes equivalent to metrizability, and local compactness, and even Čech completeness, imply paracompactness).

In the forties and at the beginning of fifties, A.A. Markov and M.I. Graev wrote a series of articles on the topological groups, especially, on free topological groups. A.A. Markov discovered that every Tychonoff space can be embedded as a closed subspace into a topological group, in fact, into the free topological group of this space [95], [96]. He invented the notions of the free topological group of a Tychonoff space X and of a free Abelian topological group of X an established the main properties of such groups. Graev established that the free topological group of a compact Hausdorff space is always the inductive limit of a (countable increasing) sequence of its compact subspaces (1948) [66]. The technique of free topological groups later proved to be very useful not only for constructing examples but also as a machinery to prove theorems (Arhangel'skii, Choban, Pestov, Tkačenko). In 1967 I offered a one year long basic course in general topology; the part 3 of this course (a whole chapter) was devoted to introduction to topological groups. In the course I proved several new facts concerning the structure of subspaces of free topological groups of compacta, in particular, the statement which was later called Joiner's Lemma. The text of notes of this course was published in 1969 in a small rotaprint edited book in Russian. M.M. Choban was attending this course, and this marks the beginning of his long road in topological algebra.

At this time I formulated a problem on topological groups which is not completely solved yet. Soon it became clear that extremal disconnectedness does not go very well with homogeneity: Frolik proved (about 1966) that every extremally disconnected homogeneous compactum is finite. Looking at another extreme case, I asked the question: does there exist (I meant, in ZFC) a nondiscrete extremally disconnected topologial group? This question remains open today. B. A. Efimov constructed in 1968 the first example of a nondiscrete extremally disconnected and homogeneous countable space [46]. A student of B.A. Efimov, Simon Sirota, partially answered my question in 1969, constructing, under the Continuum Hypothesis, an example of a nondiscrete extremally disconnected topological group [140].

In the same direction, I proved in 1967 that each compact subspace of any extremally disconnected topological group is finite [22]. Therefore, if an extremally disconnected topological group is a k-space, then it is discrete.

In sixties and seventies some important contributions to theory of compactifications were made by Russian topologists. I. I. Parovichenko wrote a fundamental paper on the structure of the Stone-Čech compactification $\beta\omega$ of the countable discrete space ω. Under Continuum Hypothesis CH, he characterizad the remainder $\beta\omega \setminus \omega$ by topological invariants, and proved that $\beta\omega \setminus \omega$ can be mapped continuously onto every compactum of weight not greater than ω_1 [108].

V. M. Ul'yanov solved in 1977 a long standing problem in the theory of compactifications. He proved that not every Hausdorff compactification of a Tychonoff space can be obtained by Wallman's method of maximal centered families of closed sets [158].

By the middle of the sixties, the theory of cardinal invariants was receiving new stimuli. It was becoming clear that one might expect especially interesting results in the theory of cardinal invariants of compact spaces. My first published paper (1959) was actually on cardinal invariants of compacta. In 1968 I went back to this topic. In 1969 I published a paper in which an approximation of the theory of dyadic compacta from positions of the theory of cardinal invariants was provided. A class of spaces very close to the class of dyadic compacta and containing all of them and stable under the operation of taking the space of all closed subsets was described. Later it turned out (L.B. Shapiro, [130]) that the class of dyadic compacta itself is not stable under the operation of taking the Vietoris exponent. In a joint paper with Ponomarev [28] we introduced the notion of tightness and obtained the first results on it. In particular, it was proved that every dyadic compactum of countable tightness is metrizable. An interesting paper on cardinal invariants of compacta, using ramification ideas, was published by Parovichenko in 1967 [109].

In 1969 I solved the Alexandroff-Urysohn problem posed in 1922, proving that the cardinality of every first countable compact Hausdorff space does not exceed 2^{ω}. The solution came to me when I visualised the notion of free sequence [23]. After many hours of thinking on the problem, I suddenly saw the solution so clearly, that when I called Alexandroff on the telephone, I confessed to him that the argument is so simple that I am afraid there might be a gap.

In the autumn of 1968 I offered to students one of my first special courses in general topology (at Moscow University), a large section of which was devoted to cardinal invariants of topological spaces. This course has attracted many undergraduate students of the first three years. Among them where Amirdjanov, Shapirovskij, Malychin, Kombarov, Ranchin, Popov. Three of them, Shapirovskij, Malychin, Ranchin, and Popov, asked me to become their scientific supervisor. This was the birth of my own seminar in general topology. It was also attended by my older students, Choban and Filippov, and by a student from Bulgaria, S.J. Nedev, who came about 1966 from Bulgaria to complete his undergraduate studies and to continue for Ph.D (called in Russia Candidate degree).

My seminar was devoted to several topics. At the center were the theory of cardinal invariants, the theory of mappings, and, to some extent, the theory of topological groups. V.I. Ponomarev also founded his seminar at the same time; among his new students were L.B. Shapiro, A.P. Kombarov, G. I. Chertanov, and G.P. Amirdjanov.

The new generation of Moscow topologists just arrived, and it did not take much time for them to demonstrate their creative potential. I must mention here that new topologists were appearing at that time not only in Moscow. In Sverdlovsk N.V. Velichko obtained his first results around 1966. His first contributions were to the theory of continuous mappings and to the theory of H-closed spaces [168]. In 1967 he constructed an example of an open continuous mapping, with compact connected fibers, of a nonmetrizable compactum onto a metrizable compact space [169]. A few years later, in 1973 or 1974, he proved that if the space of closed subsets of a Hausdorff space X is normal (in the Vietoris topology), then the space X is compact [170]. Before, this had been proved only under CH, by J. Keesling.

B.E. Shapirovskij obtained some of his beautiful results while he was still an undergraduate student of mine. In 1970 Istvan Juhasz published a paper in which he proved that Martin's Axiom, coupled with the negation of Continuum Hypothesis, implies that every perfectly normal compactum is separable (which implies, of course, that there is no Souslin line under the assumption, but that was already known). I remember that this made a lot of resonance in our seminars. I generalized this theorem proving that every sequential compact Hausdorff space with the countable Souslin number is separable under the circumstances, and asked if this could be generalized to the case of compacta of countable tighness. Shapirovskij did that in [94]. In connection with this problem I asked if every compact Hausdorff space of countable tightness is sequential. Only several years later (maybe, ten or more) I discovered that this question, without introducing explicitly the notion of tightness, was already formulated in 1956 by Mrowka and Moore.

I posed this problem as the main one to Shapirovskij (I was always giving students several problems to work on, varying in direction and in estimated difficulty). I believe, many of his results Shapirovskij obtained while trying to solve this problem, which was, apparently, waiting for Z. Balogh to come.

In 1971 Shapirovskij [132] and myself [25] proved independently that if every discrete subspace of a compact Hausdorff space is countable, then the tightness of the space is countable. Our approaches were different. I derived this conclusion from a more general theorem I established: that the tightness of any compact Hausdorff space is the supremum of the lengths of free sequences in the space [25]. This theorem later found many applications.

In particular, D.V. Ranchin in 1971 used it to prove an addition theorem for tightness: if a compact Hausdorff space X is covered by a countable family of compact subspaces of countable tightness, then the tightness of X is also countable [126].

Using the technique of free sequences, I proved in 1971 that, under CH, every nonempty sequential compact Hausdorff space is first countable at some point [24]. I asked, if this could be proved in ZFC. Only years later V.I. Malychin did construct the first consistent examples, showing that this is not the case.

However, it is still an open question, whether every homogeneous sequential compact Hausdorff space must be first countable. Under CH the answer is "yes", by the theorem above.

Tightness turned out to be one of the most important cardinal invariants, intimately related to other topological properties. In particular, tightness is instrumental in the study of Σ-products. In 1971 A. P. Kombarov established that the Σ-product of any family of Čech complete paracompact spaces of countable tightness is normal [76] (a generalization of this result for compacta see in Kombarov's paper [77]). Kombarov and Malykhin established that the tightness of the Σ-product of a family of spaces is countable if the tightness of the product of every finite subfamily of this family is countable [78].

Sequential spaces constitute an important subclass of the class of spaces of countable tightness. It is well known that a sequential compactum need not be Fréchet-Urysohn. Therefore, one has to consider the order of a sequential compactum, that is, the smallest ordinal number α such that α iterations of the sequential closure of any subset of the space give the closure of this subset. It is still an open question whether there exists in ZFC a sequential compactum of order 3 (it is easy to construct a sequential compactum of order 2). However, A. I. Bashkirov constructed a sequential compactum of order 3 with the help of Martin's Axiom [30]. He also constructed a sequential compactum of order α, for every countable ordinal number α, using Continuum Hypothesis [30].

In 1976 Shapirovskij proved that in every compact Hausdorff space the π-character does not exceed tightness [136]. It still remains unclear whether the pointwise version of this theorem is true (though the answer is "yes" under GCH).

Shapirovskij also proved that every compact Haudorff space of countable tightness has a point countable π-base [136]. One of his main results Shapirovskij obtained in 1980. He characterized compacta that can be continuously mapped onto the Tychonoff cube I^τ. A necessary and sufficient condition for that is the existence of a closed subspace with π-character (of the subspace) $\geq \tau$ at every point of it [137]. Under CH this theorem immediately implies that every compact Hausdorff space either contains a topological copy of βN or has a point of countable π-character. One of the last results of Shapirovskij is also very interesting: under CH, sequential compactness and pseudoradiality coincide for compacta.

The untimely death (August 1991) caught Shapirovskij at the height of his creative life.

In 1970 I was invited for a talk at the Mathematical Congess in Nice. Unfortunately, I was prevented from coming to the Congress, but I sent there the text of my talk in advance, and I was told by A.S. Mischenko, who went to Nice, that my lecture was presented there in my name by Mary Ellen Rudin.

The year 1970 can be considered as the starting point of a new period of development of General Topology in Moscow. Just a few years ago we were students of Alexandroff and now our own students were making name for themselves getting good new results and defending their Ph.D. thesises (kandidate dissertations).

In 1970 Choban presented his dissertation. It was on p-spaces, continuous mappings, and the exponential topology. I already mentioned some of his results.

In 1970 S.J. Nedev wrote a dissertation on "The general concepts of real-valued distance and an analyzis of topologies based on them" [98]. Extending the work of Niemytzki, he investigated systematically restrictions on distance functions (generalized metrics) and established how topological invariants of the topologies generated by these generalized metrics depend on the restrictions.

In particular, he proved that every regular symmetrizable Lindelöf space is hereditarily Lindelöf. In a joint paper, Nedev and Choban proved that every o-metrizable topological group is metrizable [99].

Nedev also proved that every regular Lindelöf first countable symmetrizable space is hereditarily separable. After the defence, Nedev returned to Bulgaria, where he started to work on selection theorems, extending the classic work of Michael (see [100]). In later years, he had excellent students of his own, like V. Valov and N. Gutev, for example.

V.V. Filippov defended his Ph.D. dissertation in 1971, on the basis of his earlier work on the theory of mappings and of p-spaces (some of this work we already mentioned above). In 1968 he introduced, independently from E. Michael, the nontrivial and important notion of a biquotient mapping. One of Fillipov's theorems in the thesis states that if f is a biquotient onto mapping of a space X with a point countable base, and every fiber under f is separable and metrizable, then Y also has a point countable base [58]. The result was new and difficult even in the case when f was a perfect mapping.

N.S. Lashnev, a student of Ponomarev, also defended his dissertation about 1970. His main theorem was remarkable: for every closed continuous mapping of a metrizable space X onto a space Y, there existsa σ-discrete subset Z of Y such that $f^{-1}(y)$ is compact, for every $y \in Y \setminus Z$ [85]. Some time earlier, I proved the same for complete metric spaces, and I thought, the completeness was essential. Here is an interesting corollary of this result of Lashnev (and of the classic theorem on metrizability of images of metrizable spaces under perfect mappings): every closed continuous image of a metrizable space is the countable union of metrizable subspaces. Again, this was known for complete metric spaces, and the general theorem was quite unexpected.

V. P. Zolotarev considered a very special type of mappings: those that are given by intersection of topologies on the same set. He proved that, for every countable ordinal α, there are two metrizable topologies on the set of natural numbers such that their intersection is a regular sequential space, the sequential order of which is α [173]. Zolotarev was a very talented young mathematician, and it was a severe blow to all of us when he perished in mountains at the beginning of the seventies.

In 1970, A.G. El'kin defended his dissertation. He was also a student of Ponomarev. The title was: "On decomposable and indecomposable spaces". One of El'kin's results was the theorem that on every dense in itself space there exists an ultrafilter such that its dense in itself elements form a base of this ultrafilter. He obtained many new results on decomposability of spaces (see [50], [51]).

In 1972 V.K. Bel'nov, a student of Ponomarev, defended a dissertation on metrizable compactifications of separable metrizable spaces [31].

Yet another of Ponomarev's students, G. I. Chertanov in his dissertation made a considerable contribution to the theory of linearly ordered spaces. He considered continuous images of products of linearly ordered spaces and generalized certain aspects of the theory of dyadic compacta [36]. Chertanov proved that every separable linearly ordered compactum can be represented as a continuous image of the "double arrow" space of Alexandroff and Urysohn. He also established that if a dyadic compactum X is an image of a linearly ordered space under a pseudo-open continuous mapping, then X is metrizable [37].

V.I. Malykhin started his life in topology at the same time as Shapirovskij. He soon became a graduate student of Ponomarev. In one of his first results Malykhin characterized countable Frechet-Urysohn spaces with a single non-isolated point in terms of closed subsets of the remainder in $\beta\omega$. His dissertation, presented in 1975, was about maximal spaces and related spaces. He got interested in my problem whether there exists a nontrivial extremally disconnected topological group. Malychin constructed, under Martin's Axiom, a nondiscrete topological group the space of which is maximal (which means that in every stronger topology on that group at least one point is isolated) [91], [92]. Of course, every maximal Hausdorff space is extremally disconnected. Malykhin also established a theorem on the algebraic structure of extremally disconnected topological groups which is now well known: every such group contains an open Abelian subgroup such that the order of each element in that subgroup is 2 [91], [92]. Countable Tychonoff spaces that do not have Hausdorff compactifications of countable tightness were constructed by Malykhin in [90].

A.V. Ostrovskij, a student of Alexandroff, submitted his Ph. D. dissertation in 1976. He considered Michael's question: is the property of being an absolute G_δ-set preserved by k-covering continuous mappings (in the class of separable metrizable spaces)? This was inspired by Hausdorff's result (1934) that open continuous mappings preserve the absolute G_δ-property (in that class of spaces). Ostrovskij solved this problem in a much more general setting and obtained many other interesting related results [104], [105].

I returned from Pakistan in the Autumn 1975. In 1977 Shapirovskij (who remained my student) presented and defended his dissertation. It is a shame that it took four meetings of the dissertation council to get the process completed. It can be very well true that the tremendous pressure and injustice Shapirovskij went through at the time were the cause of his illness and premature death. One of very interesting results in the dissertation which I did not mention so far is his theorem that density of any perfectly normal compact Hausdorff space does not exceed $\aleph_1$ [133]. See also [134] and [135].

L.B. Shapiro, a student of Ponomarev, presented his dissertation in 1977. He showed that the space $ExpX$ of closed subsets of a compact Hausdorff space X of the weight $\geq \aleph_2$ is never a dyadic compactum [130]. This is in contrast to a result of S.Sirota (1968) that if the weight of a dyadic compactum X does not exceed $\aleph_1$, then the compactum $ExpX$ is dyadic [139]. He has also proved that $ExpX$ is a Dugundji compactum if and only if X is a Dugundji compactum of the weight $\leq \aleph_1$. Shapiro established that if X is a compact Hausdorff space such that $ExpX$ is the continuous image of a Tychonoff cube I^τ, then X is metrizable [131]. His dissertation contains many other interesting results and constructions.

I. Lejbo (Pasynkov's student) introduced in his Ph. D. dissertation an interesting and useful notion of a special network. He proved [87], [88] that that the dimensions dim and Ind coincide for closed continuous images of metric spaces (generalizing the well known theorem of Morita and Katetov).

V. V. Popov (a student of mine and of V. V. Filippov) established new strong results on the space $ExpX$ of closed subsets in the Vietoris topology. He proved that if X is a perfect regular space then every compact subspace of $ExpX$ has a dense set of points of first countability. For a discrete space X he established that every compact subspace of $ExpX$ is scattered. Popov also characterized paracompact spaces X such that $ExpX$ is a k-space (see [124], [125]). Another interesting result from Popov's dissertation: the Σ-product of any family of compact Hausdorff spaces of character $\leq \omega_1$ is a k-space.

G. P. Amirdjanov (Ponomarev's student) was the first to consider the following question: when a topological space X contains a dense zero-dimensional subspace? He proved (under CH) that every Čech complete space X such that $|X| \leq 2^\omega$ contains a dense zero-dimensional subspace with a point countable base [15] and obtained other delicate results on the existence of dense subspaces with special properties (see [14], [15]).

The topic of paracompactness, its applications and generalizations, was always one of the generic themes in our seminar. Important work on it was done by S. A. Peregudov, a student of Ponomarev, at the beginning of the seventies. A family γ of open sets is called Noetherian if for every nonempty open set V the family of all elements of γ that contain V is finite.

Peregudov considered Noetherian spaces, that is, spaces which posess a base that is a Noetherian family. Clearly, every space with a uniform base is Noetherian. Peregudov established that the product of any family of Noetherian spaces is a Notherian space, and that the space of closed subsets of a Noetherian compact Hausdorff space (in the Vietoris topology) is again a Noetherian space [116]. By a slight and natural variation of the above definition, Peregudov defined strongly Noetherian spaces. This property has the advantage that it is inherited by dense subspaces. In particular, it follows that the space $C_p(X)$ of continuous realvalued functions on a space X in the topology of pointwise convergence is always Noetherian. Peregudov and Shapirovskij proved that every Noetherian compact hereditarily normal space is metrizable [117]. Peregudov applied the theory of Noetherian spaces in his systematic study of a series of paracompactness type properties lying in between metacompactness and paracompactness [115], [116].

At the beginning of the seventies Alexandroff was blessed with a new extremely bright student: E. V. Ščepin. Ščepin defined and studied the class of perfectly κ-normal spaces (introduced independently by R. Blair and T. Terada). This class of spaces contains all products of metrizable spaces and also includes all locally compact groups [148]. This lead Ščepin to definition of the important subclass of the class of perfectly κ-normal spaces, consisting of κ-metrizable spaces. It is utterly important that κ-metrizability is preserved by products and is inherited by dense subspaces [149], [150]. The class of κ-metrizable compacta was shown by Ščepin to be intimately related to the classes of Dugundji compacta and of dyadic compacta [151], [149]. Building up his theory, and starting from certain results of R. Haydon, Ščepin developed new techniques involving uncountable inverse spectra of spaces and proved a remarkable spectral theorem which allows one to prove, using transfinite induction, that certain spaces are not homeomorphic. On the basis of his spectral theorem, Ščepin solved several problems. His method was later used very efficiently by L. B. Shapiro in his study of the exponential functor.

Fedorchuk also used very effectively inverse spectra in his quest for examples of compacta with unusual combinations of cardinal invariants. Answering a question asked by I. Juhasz, he constructed, under the axiom of constructibility (or under a certain weaker assumption), a hereditarily separable compact Hausdorff space of cardinality 2^c (where $c = 2^\omega$) [54]. See also [55].

Another achievement in application of spectra is due to I. M. Kozlovskij. He proved that every metric space admits a representation by a very nice polyhedral inverse spectrum [80] (see also [79]).

I should also mention A. I. Krivoruchko who was one of the first to write a Ph. D. dissertation on cardinal invariants of function spaces in the compact-open topology. He established certain inequalities for density, cardinality and weight of function spaces [81], [82], [83]. Topology of function spaces, in the following years, became one of the main directions of research in general topology (S.P. Gul'ko, E.G. Pytkeev, O. G. Okunev, V. V. Uspenskij, myself, A. Lejderman and many others contributed to it after 1976).

I have to stop here. General topology in seventies was developing so vigorously and in so many directions that a whole new survey would be needed to describe adequately this flourishing garden. The new generation of set-theoretic topologists in Russia, which arrived at the beginning of sixties, obtained many interesting results, going in many different directions. Even a very selective survey of what was done in general topology in Russia after 1975 would more than triple this paper. And even younger topologists were still to arrive; among them my students P. A. Biryukov, M. G. Tkachenko, V.G. Pestov, O.G. Okunev, I.I. Guran, B. Bokalo, N.G. Okromeshko, V. V. Uspenskii, D. B. Shakhmatov, E. A. Reznichenko, A. V. Korovin, M. V. Matveev, D. Baturov, O. V. Sipacheva, S. Svetlichyi, V. V. Tkachuk, I. V. Yaschenko, A. S. Gul'ko, A. N. Yakivchik, R. Z. Buzyakova, A. N. Karpov. Their time, and the time of many other talented young Russian topologists, came only in the eighties and nineties.

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