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WHEN THE DIEUDONNÉ COMPLETION OF A TOPOLOGICAL GROUP IS A PARACOMPACT p-SPACE

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Abstract

We characterize topological groups G such that the Dieudonné completion μG is a topological group which is a paracompact p-space. Using this characterization, we prove that the class of such groups is closed under countable products and obtain further results on such groups and on their products. In this way we generalize some of the well known results of W.W. Comfort and K. Ross on pseudocompact groups.

0. Introduction

All spaces considered below are assumed to be Tychonoff. In this paper we study further the Dieudonné completion of a topological group. In a series of articles [16], [19], [4] it was shown that very often the Dieudonné completion of a topological group is again a topological group. The most general results in this direction one can find in [5], [6], where it was also established that there is a topological group G such that its Dieudonné completion is *not* a topological group.

The most natural question, which arises in this connection, is the following one:

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General Problem. *Suppose $\mathcal{G}$ is a class of Dieudonné complete topological groups. Characterize topological groups G such that the Dieudonné completion of G belongs to $\mathcal{G}$.*

The well known theorem of W.W. Comfort and K.A. Ross from [9] is the first, and very beautiful, result in this direction. It tells us when the Dieudonné completion μG of a topological group G is a compact group: this happens precisely if G is pseudocompact. Here we prove a parallel result characterizing topological groups G such that the Dieudonné completion μG is a topological group which is a paracompact p-space. In fact, the theorem we obtain easily implies the result of Comfort and Ross.

An important class of topological spaces is the class of *p-spaces* (or *feathered* spaces (see [1, 2]). This class contains all metrizable spaces as well as all locally compact spaces, and even all Čech-complete spaces. Much more general is the class of spaces of point-countable type [8].

A space X is *of point-countable type* at a point $x \in X$, if there exists a compact subset F of X with a countable base of neighbourhoods in X such that $x \in F$. A space X is said to be of *point-countable type*, if it is of point-countable type at each point x of X.

Since the classes of compact and locally compact topological groups play a very important role in topological algebra and its applications, it was natural to consider topological groups which are p-spaces. An argument on some basic properties of such groups was already included in the paper [2], where p-spaces were introduced. A more complete survey of this class of groups and their role can be found in [14], where they are also called *almost metrizable groups*. Here we just mention that a topological group is a p-space if and only if it is of point-countable type, and that a topological group G is a p-space if and only if there exists a homomorphism h of G onto a metrizable group M which is a perfect mapping of the space G onto the space M (these results are not difficult to prove).

Note, that every feathered topological group is paracompact, by the result above (see [2], [8]). Clearly, a topological group G is feathered if and only if there exists a nonempty compact subset F of G with a countable base of neighbourhoods (see [14]).

A point a of a space X is called *a pseudocompactness point* of X, if there exists a sequence $\{U_n : n \in \omega\}$ of open neighbourhoods of a, satisfying the condition: for every sequence $\{V_n : n \in \omega\}$ of non-empty open sets such that $V_n \subset U_n$ for each $n \in \omega$, there exists a point of accumulation in X. This notion was introduced in [4].

A space X is said to be *pointwise pseudocompact*, if each point of X is a pseudocompactness point. Clearly, in a pseudocompact space every point is a pseudocompactness point and, therefore, every pseudocompact space is pointwise pseudocompact.

According to E.Michael (see [13]), a point x of a space X is said to be *a q-point*, if there exists a sequence $\{U_n : n \in \omega\}$ of open neighbourhoods of x satisfying the following condition:

(q) *for every sequence $\{x_n : n \in \omega\}$ of points in X such that $x_n \in U_n$ for each $n \in \omega$, there exists a point of accumulation in X.*

A space is called *a q-space* if all its points are q-points. Obviously, we have:

Proposition 0.1. *Every q-point of a space X is a pseudocompactness point of it, and, therefore, every q-space is pointwise pseudocompact.*

Corollary 0.2. *Every space of point-countable type is pointwise pseudocompact.*

Proof. Since each space X of point-countable type is a q-space [13], this follows from Proposition 0.1. $\square$

1. The Dieudonné Completion of a Pointwise Pseudocompact Topological Group

Every topological group can be represented as a dense subgroup of its Rajkov completion ρG, which is a topological group complete with respect to the two-sided uniformity [14]. Here is our first result:

Theorem 1.1. *If G is a pointwise pseudocompact topological group, then its Rajkov completion ρG is also a pointwise pseudocompact topological group.*

Theorem 1.1 obviously follows from the next more general statement.

Theorem 1.2. *If a topological group G contains a dense pointwise pseudocompact subspace Y, then G is pointwise pseudocompact.*

It is clear that Theorem 1.2, in its turn, follows from the next two obvious facts:

Proposition 1.3. *If Y is a dense subspace of a space X, and y is a pseudocompactness point of Y, then y is also a pseudocompactness point of X.*

Proposition 1.4. *If a homogeneous space X contains a dense pointwise pseudocompact subspace Y, then X is also pointwise pseudocompact.*

We can improve considerably on Theorem 1.1. This will lead us to the main results of this section. First, we present some technical statements that we need.

Recall that a subset K of a space X is said to be *R-bounded*, or just *bounded* in X, if every continuous function on X is bounded on K. The following fact is a part of the folklore; we present its proof for the sake of completeness.

Proposition 1.5. *The closure of any bounded subset A of a Dieudonné complete space X is compact.*

Proof. We may assume that A is closed in X, since the closure of a bounded subset is, obviously, closed. Let us show that A is also closed in the Stone-Čech compactification βX of X. Assume the contrary. Then we can fix a point $z \in \beta X \setminus X$ such that z is in the closure of A in βX. Since X is Dieudonné complete, there exists a locally finite open covering γ of X such that z is not in the closure of U, for each $U \in \gamma$ [11]. Since A is bounded in X, the family η of all $U \in \gamma$ such that $U \cap A$ is not empty is finite (this is well known and easy to prove, see [3]). Since $A \subset \cup\eta$ and $\eta \subset \gamma$, from the choice of γ it follows that z is not in the closure of $\cup\eta$. Therefore, z is not in the closure of A, a contradiction. □

Proposition 1.6. *Suppose x is a pseudocompactness point of X, and let $\xi = \{U_n : n \in \omega\}$ be a sequence of open neighbourhoods of x, satisfying the condition: for every sequence $\{V_n : n \in \omega\}$ of non-empty open sets such that $V_n \subset U_n$, for each $n \in \omega$, there exists a point of accumulation in X. Suppose also that the space X is Dieudonné complete. Then the space X is of point-countable type at x.*

Proof. Since X is regular, we may assume that $\overline{U_{n+1}} \subset U_n$, for each $n \in \omega$. Then the set $P = \cap\{U_n : n \in \omega\}$ is closed in X. From the main assumption about ξ it obviously follows that P is bounded in X. Therefore, P is compact, by Proposition 1.5, since X is Dieudonné complete.

Let us show that ξ is a base of the family $\mathcal{F}$ of all open neighbourhoods of the set P in X. Assume the contrary. Then, obviously, we can fix an open set V such that $P \subset V$ and $U_n \setminus \overline{V} \neq \emptyset$, for each $n \in \omega$. Put $W_n = U_n \setminus \overline{V}$, for each $n \in \omega$. By the main assumption on the family ξ, there exists a point of accumulation y for the family $\eta = \{W_n : n \in \omega\}$ in X. It is clear, that y belongs to the closure of U_n, for each $n \in \omega$. It follows that y is in P. However, this is impossible, since y is, obviously, in $X \setminus V$ and $P \subset V$. The contradiction shows that ξ is a base of the set P in X. □

Now from Proposition 1.6 and Corollary 0.2 we obtain:

Theorem 1.7. *A Dieudonné complete space X is pointwise pseudocompact if and only if it is of point-countable type.*

Propositions 1.4 and 1.6 together imply the next result:

Corollary 1.8. *If a Dieudonné complete homogeneous space X contains a dense pointwise pseudocompact subspace Y, then X is a space of point-countable type.*

Now from Theorem 1.1 and Corollary 1.8 we have:

Theorem 1.9. *The Rajkov completion ρG of every pointwise pseudocompact topological group G is a feathered topological group (that is, a paracompact p-space).*

Proof. Since ρG is Dieudonné complete, from Theorem 1.1 and Corollary 1.8 it follows that ρG is a space of point-countable type. However, every topological group of point-countable type is a paracompact p-space (see [14]). $\square$

The converse to Theorem 1.9 does not hold, since not every totally bounded topological group is paracompact, while the Rajkov completion of each totally bounded topological group is compact [14]. The next result is more symmetric.

Theorem 1.10. *A topological group G is pointwise pseudocompact if and only if its Dieudonné completion μG is a feathered topological group.*

Proof. Necessity.

We need the following result from [4]:

Theorem A. *Every pointwise pseudocompact topological group is a Moscow space.*

Recall that a space X is *Moscow* if, for every open set U in X, the closure of U is the union of G_δ-subsets of X. See about Moscow spaces [5], [6], and [7]. One of the key results on Moscow topological groups is the next one [5]:

Theorem B. *For every Moscow topological group G, the Dieudonné completion μG is a topological group.*

Now from Theorem 1.1 and Corollary 1.8 it follows that μG is a space of point-countable type. However, every topological group of point-countable type is a paracompact p-space.

Sufficiency. This follows from the next general statement:

Theorem 1.11. *Every G_δ-dense subspace X of a feathered topological group G is pointwise pseudocompact.*

Proof. Take any point $x \in X$. Since G is a space of point-countable type, x belongs to a compact subspace F of G such that there exists a countable base $\{V_n : n \in \omega\}$ of neighbourhoods of F in G. Put $P = F \cap X$ and $U_n = V_n \cap X$, for each $n \in \omega$. We, obviously, have $x \in P \subset U_n$, for each $n \in \omega$.

Now take any sequence $\xi = \{W_n : n \in \omega\}$ of non-empty open sets in X such that $W_n \subset U_n$ for each $n \in \omega$. Let us show that there exists a point of accumulation of ξ in X.

Since X is dense in G, there exists an open subset O_n of G such that $O_n \cap X = W_n$ and $O_n \subset V_n$. Since F is compact, and $\{V_n : n \in \omega\}$ is a base of open neighbourhoods of F in G, some point y of F is an accumulation point of the sequence $\eta = \{O_n : n \in \omega\}$ in G. Put $E_k = \cup\{O_n : k \leq n\}$, for $k \in \omega$. Then, clearly, $y \in \overline{E_k}$, for each $k \in \omega$. By Theorem A, the space G is Moscow. Since every set E_k is open in G, it follows from Theorem A that there exists a G_δ-subset B of G such that $y \in B \subset F$ and every point z of B is an accumulation point of η (note, that F is also a G_δ-subset of G). However, X is G_δ-dense in G. It follows that $B \cap X \neq \emptyset$. Obviously, every point of $B \cap X$ is an accumulation point of the sequence $\xi = \{W_n : n \in \omega\}$. Hence, X is pointwise pseudocompact. The proof of Theorem 1.10 is complete. $\square$

In fact, for any Moscow topological group G, its Dieudonné completion μG is just the G_δ-closure $\rho_\omega G$ of G in the Rajkov completion of G (see [6]) (which is obviously a topological group). Therefore, in view of Theorem 1.11, we can reformulate Theorem 1.10 as follows:

Theorem 1.12. *A topological group G is pointwise pseudocompact if and only if its G_δ-closure $\rho_\omega G$ in the Rajkov completion of G is a feathered topological group.*

Though the above results perfectly generalize several well known statements from the theory of compact groups, they do not cover the next curious fact from Comfort-Ross theory [9]: if, for a topological group G, μG is just a compact space, then μG is a compact topological group. Trying to extend this result, we arrive at the next questions. Suppose G is a topological group such that μG is a paracompact p-space. Is then μG a topological group? Is then G pointwise pseudocompact? Both questions get positive answers below.

Recall that a subspace Y of a space X is said to be C-*embedded* in X if every continuous real-valued function on Y can be extended to a continuous function on X. It is well known that a space X is always C-embedded in μX [13, 12].

Lemma 1.13. *Suppose Y is a dense C-embedded subspace of a space X, and suppose that x^* is a pseudocompactness point of X. Then:*

1) There is a G_δ-set F in X such that $x^* \in F$ and every point of F is a pseudocompactness point of X;

2) If $x^* \in Y$, then x^* is a pseudocompactness point of Y;

3) There exists a pseudocompactness point of X in the space Y.

Proof. Fix a sequence of open neighbourhoods $\xi = \{W_n : n \in \omega\}$ of x^* in X that witnesses that x^* is a pseudocompactness point of X. Clearly, we may assume that the closure of W_{n+1} in X is contained in W_n, for each $n \in \omega$. Put $U_n = W_n \cap Y$ and $F = \cap\xi = \cap\{W_n : n \in \omega\}$. It is obvious that every point of F is a pseudocompactness point of X. This proves 1). Now assume that x^* is in Y. Put $P = F \cap Y$ and $U_n = W_n \cap Y$, for each $n \in \omega$. We claim that the sequence $\eta = \{U_n : n \in \omega\}$ witnesses that x^* is a pseudocompactness point of Y.

Assume the contrary. Then there exists a discrete in Y sequence $\{V_n : n \in \omega\}$ of non-empty open sets in Y such that $V_n \subset U_n$, for each $n \in \omega$. Fix $y_n \in V_n$, for each $n \in \omega$. Obviously, there exists a continuous real-valued function f on Y such that $f(y_n) > n$, for $n \in \omega$. Since Y is C-embedded in X, this function f can be extended to a continuous real-valued function f^* on X. Then $f^*(y_n) = f(y_n) > n$, for $n \in \omega$. Since $y_n \in W_n$, we can choose an open neighbourhood $O(y_n)$ of y_n in X such that $O(y_n) \subset W_n$ and $f^*(x) > n$, for each $x \in O(y_n)$. However, from the choice of $\xi = \{W_n : n \in \omega\}$ and F it follows that the sequence $\{O(y_n) : n \in \omega\}$ has a limit point z in $F \subset X$. Clearly, f^* is unbounded on any neighbourhood of z; therefore, f^* is discontinuous at z, a contradiction. This proves 2).

To prove 3), we observe that Y is G_δ-dense in X, since Y is dense and C-embedded in X (this is almost obvious; see [12]). Therefore, $P = F \cap Y$ is nonempty in any case, and every point of P is a pseudocompactness point of X. Now from 2) it follows that every point of P is a pseudocompactness point of Y. Thus, 3) is proved. $\square$

From Lemma 1.13 we immediately obtain:

Theorem 1.14. *If Y is a dense C-embedded subspace of a pointwise pseudocompact space X, then Y is pointwise pseudocompact.*

Corollary 1.15. *Every dense C-embedded subspace of a space of point countable type is pointwise pseudocompact.*

However, the main result we obtain with the help of Lemma 1.13 is the following theorem.

Theorem 1.16. *Let G be a topological group. Then the following conditions are equivalent:*

1) The Dieudonné completion μG of G contains a nonempty compact subspace F with a countable base of open neighbourhoods of F;

2) μG is a space of point countable type;

3) μG is a paracompact p-space;

4) μG is a topological group which is a paracompact p-space;

5) The space G is pointwise pseudocompact.

Proof. It is enough to refer to Theorem 1.10 and to Lemma 1.13. □

2. Some Corollaries

The results in the previous section allow to partially generalize the Comfort-Ross theorem on preservation of pseudocompactness under products in the class of topological groups. Indeed, we have:

Theorem 2.1. *Let G_i be a feathered topological group, and Y_i a C-embedded dense subspace of G_i, for each $i \in \omega$. Then the product space $Y = \Pi\{Y_i : i \in \omega\}$ is C-embedded in the product space $G = \Pi\{G_i : i \in \omega\}$.*

Proof. Indeed, Y_i is G_δ-dense in G_i, for $i \in \omega$. It follows that Y is G_δ-dense in G. Obviously, G is a feathered topological group, since the product of any countable family of feathered spaces is a feathered space (see [1, 2]). By Theorem A, G is a Moscow space. Therefore, Y is C-embedded in G. □

Theorem 2.2. *Let G_i be a pointwise pseudocompact topological group, for each $i \in \omega$, and $G = \Pi\{G_i : i \in \omega\}$ the product. Then the space G is also pointwise pseudocompact.*

Proof. Clearly, $\rho_\omega G = \Pi\{\rho_\omega(G_i) : i \in \omega\}$. By Theorem 1.9, each $\rho_\omega(G_i)$ is a feathered topological group. Therefore, their product $\rho_\omega G$ is also a feathered topological group. Note that G is G_δ-dense in $\rho_\omega G$. Applying Theorem 1.11, we conclude that the topological group G is also pointwise pseudocompact. □

Example 2.3. Theorem 2.2 does not generalize to the case of an uncountable family of factors. Indeed, take the topological group $G = Z^{\omega_1}$, where Z is the usual discrete group of integers. Then Z is pointwise pseudocompact, while G is not pointwise pseudocompact, since G is Dieudonné complete and G is not of point countable type. However, we have the next result for any number of pointwise pseudocompact factors:

Theorem 2.4. *The product of any family $\xi = \{G_\alpha : \alpha \in A\}$ of pointwise pseudocompact topological groups is a Moscow group.*

Proof. To prove this theorem, we will establish several intermediate results, some of them of independent interest. One of them involves the notion of o-tightness introduced by M.G. Tkačenko [18].

We say that the *o-tightness* of a space X is countable (and write $ot(X) \leq \omega$) if for every family γ of open sets in X and for every point x in the closure of $\cup\gamma$, there exists a countable subfamily η of γ such that $x \in \overline{\cup\eta}$. □

It is well known that the Souslin number of every pseudocompact topological group is countable. Unfortunately, we can not expect the same to be true for all pointwise pseudocompact topological groups, since every discrete group is in this class. However, we have the following curious result which will provide us with an interesting corollary.

Theorem 2.5. *If a topological group G is pointwise pseudocompact, then the o-tightness of G is countable.*

Proof. By Theorem 1.10, G is a dense subspace of a feathered topological group G^*. Every p-space is a k-space [8]; therefore, by a deep theorem of Tkačenko [16], the o-tightness of G^* is countable. Since G is dense in G^*, it follows that the o-tightness of G is countable. □

We continue the proof of Theorem 2.4. By Theorem 2.2 and Theorem A, the product of any finite subfamily of the family ξ is a Moscow group of countable o-tightness. Now it follows from Theorem 2.27 in [7] that the product of all spaces in ξ is a Moscow topological group. □

Combining Theorem 2.4 with one of the main results in [5], we obtain:

Corollary 2.6. *For any family $\xi = \{G_\alpha : \alpha \in A\}$ of pointwise pseudocompact topological groups such that the Souslin number of the product $G = \Pi\{G_\alpha : \alpha \in A\}$ is Ulam non-measurable, we have:*

$$\mu G = \Pi\{\mu(G_\alpha) : \alpha \in A\}.$$

Theorem 2.4 can be proved using an alternative approach, via κ-metrizable spaces (see definition in [15]).

Theorem 2.7. *Every pointwise pseudocompact topological group G is κ-metrizable.*

Proof. Indeed, ρG is a feathered topological group, by Theorem 1.1. Then, by a result of E. Sčepin (see [15], Corollary 2a), the space ρG is κ-metrizable. Since G is dense in ρG, it follows that G is κ-metrizable [15]. □

Corollary 2.8. *The product of any family of pointwise pseudocompact topological groups is a κ-metrizable topological group. Hence, this product group is a Moscow group.*

Let us apply our technique to obtain some results on locally bounded topological groups. Recall that a space X is is said to be *locally bounded*, if it can be covered by open bounded in X subsets. The next statement is obvious:

Proposition 2.9. *Every locally bounded space is pointwise pseudocompact.*

Proposition 2.10. *Let G be a topological group, and let Y be a G_δ-dense subspace of G. Then the next three conditions are equivalent:*

a) G is locally bounded;

b) Y is locally bounded;

c) There exists a non-empty subset W of Y which is open in Y and bounded in G.

Moreover, if at least one of the conditions a) - c) is satisfied, then Y is C-embedded in G.

Proof. Clearly, b) implies c). Since Y is dense in G, a) implies c). Let us show that c) implies a). Take any non-empty open subset V of Y such that V is bounded in G. Then $\overline{V}$ is bounded in G (where the closure is taken in G). Since Y is dense in G, $\overline{V}$ contains a non-empty open subset U of G. Obviously, U is bounded in G. Since G is topologically homogeneous, it follows that the space G is locally bounded. Notice, that we have not used the G_δ-denseness of Y in G so far. We are going to use it now to show that a) implies b).

Indeed, every locally bounded space is pointwise pseudocompact. Therefore, given a), Y is C-embedded in G by Theorems A and B.

Now take any $y \in Y$. Since G is locally bounded, there exists an open neighbourhood U of y in G such that U is bounded in G. Then $V = U \cap Y$ is a non-empty open subset of Y bounded in G. Since Y is C-embedded in G, it follows that V is bounded in Y.

The last assertion in Proposition 2.10 follows from Theorems A and B, as we just saw it in the argument above. □

Proposition 2.10 is closely related to the next result of Comfort and Trigos–Arrieta [10]:

Proposition 2.11. *Let G be a topological group, and let Y be a dense subgroup of G. Then the next assertions are equivalent:*

a) Y is locally bounded;

b) G is locally bounded, and Y is G_δ-dense in G.

Proof. By Proposition 2.10, b) implies a). Assume that Y is locally bounded. Then, as it was shown in the proof of Proposition 2.10, G is locally bounded. Now take any non-empty open subset V of the space Y such that V is bounded in Y. Let Z be the G_δ-closure of Y in G. Since Y is a subgroup of G, it follows that Z is a subgroup of G. Since V is bounded in Y, the closure $\overline{V}$ of V in G is contained in Z, by the standard reasoning. But $\overline{V}$ contains a non-empty open subset of G, since G is regular and Y is dense in G. Therefore, Z contains a non-empty open subset of G. It follows that Z is an open subgroup of G. This implies that Z is closed in G. Since Z contains Y, Z is dense in G. Hence $Z = G$. $\square$

Every Rajkov complete locally bounded topological group is locally compact, since the closure of any bounded subset in a Dieudonné complete space is compact. This observation, combined with Propositions 2.10 and 2.11, leads us to the following conclusion:

Corollary 2.12. [12] *For every locally bounded topological group G, the Rajkov completion of G is a locally compact group in which G is C-embedded, and therefore, G_δ-dense.*

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