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CLOSED LOCALLY PATH-CONNECTED SUBSPACES OF FINITE-DIMENSIONAL GROUPS ARE LOCALLY COMPACT

TARAS BANAKH AND LYUBOMYR ZDOMSKYY

Abstract. We prove that each closed locally continuum-connected subspace of a finite dimensional topological group is locally compact. This allows us to construct many 1-dimensional metrizable separable spaces that are not homeomorphic to closed subsets of finite-dimensional topological groups, which answers in the negative a question of Dmitriĭ Shakhmatov. Another corollary is a characterization of Lie groups as finite-dimensional locally continuum-connected topological groups. For locally path connected topological groups this characterization was proved by Andrew M. Gleason and Richard S. Palais in 1957.

1. Introduction

It follows from the classical Menger-Nöbeling-Pontryagin Theorem that each separable metrizable space X of dimension $n = \dim X < \infty$ admits a topological embedding $e : X \to G$ into a metrizable separable group G of dimension $\dim(G) = 2n + 1$ (for such a group G, we can take the $(2n + 1)$-dimensional Euclidean space $\mathbb{R}^{2n+1}$). In [15] (see also [4, Question 7]), Dmitriĭ Shakhmatov asked if we can additionally require of $e : X \to G$ to be a **closed** embedding? In this paper we shall give a strongly negative answer to this question constructing simple 1-dimensional spaces that are

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