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by

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DEFINING TOPOLOGIES ON TREES

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Abstract. In this paper we collect results concerning topologies defined on trees. We also introduce the new concept of a topology on a tree generated by a collection of filters. Some applications are presented as well.

1. Definitions and basic construction

For a partial order $(X, \leq)$ and $x \in X$ we shall use the following notation:

$$[x, \rightarrow) = \{z \in X : x \leq z\},$$
$$(\leftarrow, x) = \{z \in X : z < x\},$$
$$(x, \rightarrow) = \{z \in X : x < z\},$$
$$[x, y] = \{z \in X : x \leq z \leq y\}.$$

A *tree* is a partial order $(T, \leq)$ such that:

(1) there exists the least element (the root) in T,
(2) for every $t \in T$ the set $(\leftarrow, t)$ is well ordered by the relation $\leq$.

Clearly, for every $s \in T$ the set $[s, \rightarrow)$ is a tree in which the element s is the root. As usual, the order type of the set $(\leftarrow, t)$ is called the height of t in T and denoted by $\mathrm{ht}(t, T)$.

If $(T, \leq)$ is a tree and $t \in T$ then

$$\mathrm{succ}_T(t) = \{s \in T : s \text{ is minimal in } (t, \rightarrow)\}$$

denotes the set of all *immediate successors* of the element t. If the tree T is fixed we shall write $\mathrm{succ}(t)$ instead of $\mathrm{succ}_T(t)$.

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