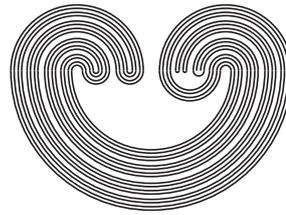


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## DEFINING TOPOLOGIES ON TREES

by

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## DEFINING TOPOLOGIES ON TREES

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**ABSTRACT.** In this paper we collect results concerning topologies defined on trees. We also introduce the new concept of a topology on a tree generated by a collection of filters. Some applications are presented as well.

### 1. DEFINITIONS AND BASIC CONSTRUCTION

For a partial order  $(X, \leq)$  and  $x \in X$  we shall use the following notation:

$$\begin{aligned} [x, \rightarrow) &= \{z \in X : x \leq z\}, \\ (\leftarrow, x) &= \{z \in X : z < x\}, \\ (x, \rightarrow) &= \{z \in X : x < z\}, \\ [x, y] &= \{z \in X : x \leq z \leq y\}. \end{aligned}$$

A *tree* is a partial order  $(T, \leq)$  such that:

- (1) there exists the least element (the root) in  $T$ ,
- (2) for every  $t \in T$  the set  $(\leftarrow, t)$  is well ordered by the relation  $\leq$ .

Clearly, for every  $s \in T$  the set  $[s, \rightarrow)$  is a tree in which the element  $s$  is the root. As usual, the order type of the set  $(\leftarrow, t)$  is called the height of  $t$  in  $T$  and denoted by  $\text{ht}(t, T)$ .

If  $(T, \leq)$  is a tree and  $t \in T$  then

$$\text{succ}_T(t) = \{s \in T : s \text{ is minimal in } (t, \rightarrow)\}$$

denotes the set of all *immediate successors* of the element  $t$ . If the tree  $T$  is fixed we shall write  $\text{succ}(t)$  instead of  $\text{succ}_T(t)$ .

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