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VORONOI TESSELLATIONS FOR MATCHBOX MANIFOLDS

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Abstract. Matchbox manifolds $\mathfrak{M}$ are a special class of foliated spaces, which includes as special examples exceptional minimal sets of foliations, weak solenoids, suspensions of odometer and Toeplitz actions, and tiling spaces associated to aperiodic tilings with finite local complexity. Some of these classes of examples are endowed with an additional structure, that of a transverse foliation, consisting of a continuous family of Cantor sets transverse to the foliated structure. The purpose of this paper is to show that this transverse structure can be defined on all minimal matchbox manifolds. This follows from the construction of uniform stable Voronoi tessellations on a dense leaf, which is the main goal of this work. From this we define a foliated Delaunay triangulation of $\mathfrak{M}$, adapted to the dynamics of $\mathcal{F}$. The result is highly technical, but underlies the study of the basic topological structure of matchbox manifolds in general. Our methods are unique in that we give the construction of the Voronoi tessellations for a complete Riemannian manifold L of arbitrary dimension, with stability estimates.

1. Introduction and Main Theorems

An *n-dimensional foliated space* $\mathfrak{M}$ is a continuum locally homeomorphic to a product of a disk in $\mathbb{R}^n$ and a Hausdorff separable topological space. The leaves of the foliation $\mathcal{F}$ of $\mathfrak{M}$ are the maximal connected components with respect to the fine topology on $\mathfrak{M}$ induced by the plaques of the local product structure. A *matchbox manifold* is a foliated space such that the local transverse models are totally disconnected, and the leaves have a smooth structure. Thus, a matchbox manifold is a continuum $\mathfrak{M}$ whose arc-components define a smooth foliated structure on $\mathfrak{M}$. A matchbox manifold is *minimal* if every leaf of $\mathcal{F}$ is dense in $\mathfrak{M}$.

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