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THE GROUPS S^3 AND $\mathsf{SO}(3)$ HAVE NO INVARIANT BINARY k-NETWORK

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Abstract. A family $\mathcal{N}$ of closed subsets of a topological space X is called a *closed k-network* if for each open set $U \subset X$ and a compact subset $K \subset U$ there is a finite subfamily $\mathcal{F} \subset \mathcal{N}$ with $K \subset \bigcup \mathcal{F} \subset U$. A compact space X is called *supercompact* if it admits a closed k-network $\mathcal{N}$ which is *binary* in the sense that each linked subfamily $\mathcal{L} \subset \mathcal{N}$ is centered. A closed k-network $\mathcal{N}$ in a topological group G is *invariant* if $xAy \in \mathcal{N}$ for each $A \in \mathcal{N}$ and $x, y \in G$. According to a result of Kubiś and Turek [3], each compact (abelian) topological group admits an (invariant) binary closed k-network. In this paper we prove that the compact topological groups S^3 and $\mathsf{SO}(3)$ admit no invariant binary closed k-network.

1. Introduction

In this note we shall discuss the problem of the existence of invariant binary k-networks for compact G-spaces and compact topological groups.

A family $\mathcal{A}$ of subsets of a set X is called

- *linked* if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{A}$;
- *centered* if $\cap \mathcal{F} \neq \emptyset$ for any finite subfamily $\mathcal{F} \subset \mathcal{A}$;
- *binary* if each linked subfamily of $\mathcal{F}$ is centered.

A family $\mathcal{A}$ of subsets of a topological space X is called a *k-network* if for any open set $U \subset X$ and a compact subset $K \subset U$ there is a finite subfamily $\mathcal{F} \subset \mathcal{A}$ with $K \subset \bigcup \mathcal{F} \subset U$, see [2, §11]. If each set $A \in \mathcal{A}$ of a k-network is closed in X, then $\mathcal{A}$ will be called a *closed k-network*.

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