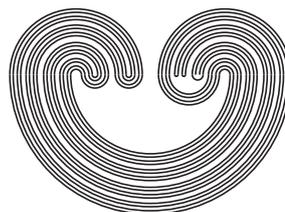

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DYNAMICS OF DISCONTINUOUS MAPS VIA CLOSED RELATIONS

by

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DYNAMICS OF DISCONTINUOUS MAPS VIA CLOSED RELATIONS

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ABSTRACT. For the dynamics of a discontinuous map on a compact metric space, we describe an approach using suitable closed relations and connect it with the continuous dynamics on an invariant G_δ subset and with the continuous dynamics on the compact space of sample paths.

1. INTRODUCTION

There has been some interest in extending the theory of dynamical systems to discontinuous maps with special focus on quasi-continuous maps. See, for example, Crannell and Martelli (2000). A subset A of a metric space X is called *quasi-open* when $A \subset \overline{A^\circ}$. That is, A is an open set together with part of its topological boundary. A map $f : X_1 \rightarrow X_2$ between metric spaces is called *quasi-continuous* when the pre-image of every open subset of X_2 is at least quasi-open.

As might be expected this leads to some oddities which, in my opinion, are best handled by using the existing theory as extended to closed relations, e.g. as in Akin (1993), but with a bit of trimming. Let me illustrate with an example of maps on $I = [0, 1]$.

$$(1.1) \quad f_0(x) = \begin{cases} \frac{1}{2} - x & 0 \leq x \leq \frac{1}{2}, \\ \frac{3}{2} - x & \frac{1}{2} < x \leq 1. \end{cases}$$

$$f_1(x) = \begin{cases} \frac{1}{2} - x & 0 \leq x < \frac{1}{2}, \\ \frac{3}{2} - x & \frac{1}{2} \leq x \leq 1. \end{cases}$$

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Key words and phrases. Discontinuous Map, Quasi-Continuous Map, Almost One-to-One Map, Irreducible Map, Closed Relation, Almost Open Map, Suitable Relation.

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