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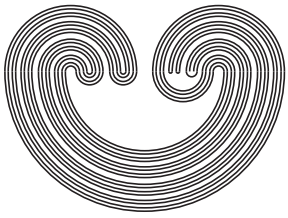

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Nonpositively Curved Manifolds Containing a Prescribed Nonpositively Curved Hypersurface

by

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NONPOSITIVELY CURVED MANIFOLDS CONTAINING A PRESCRIBED NONPOSITIVELY CURVED HYPERSURFACE

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Abstract. We use pinched smooth hyperbolization to show that every closed, nonpositively curved n-dimensional manifold M can be embedded as a totally geodesic submanifold of a closed, nonpositively curved $(n+1)$-dimensional manifold $\widehat{M}$ of geometric rank one.

Ralf Spatzier asked the author the following interesting question: " For a closed manifold M with sectional curvature ≤ 0 (e.g., a closed, nonpositively curved, locally symmetric manifold), is there a closed manifold $\widehat{M}$ of one dimension higher with sectional curvature ≤ 0 and which has geometric rank 1 (and thus is not a product) that contains M as a totally geodesic submanifold?" The answer to this question is yes, thanks to recent technology of pinched smooth hyperbolization [4]. In this paper we give a construction of such a manifold $\widehat{M}$.

Theorem 1. *Let (M, g_M) be a closed, Riemannian manifold of dimension n with sectional curvature $\kappa(M) \leq 0$. There exists a closed, Riemannian $(n+1)$-dimensional manifold $\widehat{M}$ of geometric rank 1 with sectional curvature $\kappa(\widehat{M}) \leq 0$ and an isometric embedding $f\colon M \longrightarrow \widehat{M}$.*

Proof. Let $\triangle$ be a smooth triangulation of M. We extend $\triangle$ to a triangulation of $M \times [0,1]$. We cone off the boundary of $M \times [0,1]$ (which has two components) and denote the resulting simplicial complex by X. Then X is a manifold with one singular cone point $*$; that is, $X \setminus \{*\}$ is a manifold. Let $h(X)$ be a strict hyperbolization of X [2]. Then $h(X)$ is a

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