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by

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NOTES ON THE OD-LINDELÖF PROPERTY

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Abstract. A space is od-compact (od-Lindelöf, respectively) provided any cover by open dense sets has a finite (countable, respectively) subcover. We first show with simple examples that these properties behave quite poorly under finite or countable unions. We then investigate the relations between Lindelöfness, od-Lindelöfness, and linear Lindelöfness (and similar relations with "compact"). We prove, in particular, that if a T_1 space is od-compact, then the subset of its non-isolated points is compact. If a T_1 space is od-Lindelöf, we only get that the subset of its non-isolated points is linearly Lindelöf, though Lindelöfness follows if the space is moreover locally openly Lindelöf (i.e., each point has an open Lindelöf neighborhood).

1. Introduction

In the middle of an argument involving Baire theorem, we noticed that we did not need the space under scrutiny to be really Lindelöf, but rather that any cover of it by open *dense* sets had a countable subcover. We then wondered whether this alternative definition of Lindelöfness, called here *od-Lindelöfness*, was interesting in itself, as well as the similarly defined notion of od-compactness. These notes are the results of our musings, which may be summarized as follows.

• od-compact spaces behave quite horribly when taking unions, even when just two subspaces are involved and there are even completely metrizable spaces that behave badly in this respect. A finite union of od-compact *closed* spaces is od-compact, though. On the other hand, a countable union of od-Lindelöf closed spaces does not need to be od-Lindelöf.

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