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PERIODS OF ENDOMORPHISMS ON AN ABELIAN GROUP

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ABSTRACT. Given a group G and an endomorphism f of G, a point $x \in G$ is a periodic point of f of period $n \in \mathbb{N}$ if $f^n(x) = x$ and $f^k(x) \neq x$ for all $1 \leq k < n$. The set of periods of f is the set $Per(f) = \{n \in \mathbb{N} : f$ has a periodic point of period $n\}$. In this paper, we characterize the sets of periods of an endomorphism of an abelian group. We prove that $\{A \subset \mathbb{N} : 1 \in A$ and A is closed under lcm$\}$ is the family of period sets of endomorphisms of an abelian group.

1. INTRODUCTION

A dynamical system is a pair (X, f) consisting of a topological space X and a continuous self map f on it. We denote f^n the composition of f with itself $n-1$ times. For $x \in X$, the sequence x, $f(x)$, $f^2(x)$,..., $f^n(x)$,... is called the *forward f-trajectory* of x and the set $\{f^n(x) : n = 0, 1, 2,...\}$ is called the *forward f-orbit* of x. A point $x \in X$ is a *periodic point* of f of period (f-period) $p \in \mathbb{N}$ if $f^p(x) = x$ and $f^m(x) \neq x$ for all $1 \leq m \leq p-1$. The periodic points of period 1 are called *fixed* points of f. Let $Per(f)=$ $\{n \in \mathbb{N}$ such that f has a point of period $n\}$ and we call this the *set of periods* (or simply *period set*) of f. All these notions make sense for any set not necessarily a topological space and for any self map not necessarily continuous. For $m, n \in \mathbb{N}$, $m \vee n$ denotes the least common multiple (lcm) of m and n; for $A, B \subset \mathbb{N}$, $A \bigvee B = \{m \vee n : m \in A, n \in B\}$; and a triple $(k_1, k_2, k_3) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ satisfies *property P* if each number divides the

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