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PAUL BANKSTON

Abstract. An interpretation of betweenness on a set is *gap free* if each two distinct points of the set have a third point between them. In this paper we are interested in gap free betweenness relations naturally induced by the topology of Hausdorff continua. In particular, we say *c lies between a and b in the K-interpretation* precisely when every subcontinuum that contains both a and b also contains c. We explore the connection between K-gap freeness and hereditary unicoherence.

1. Introduction

If $[\ ,\ ,\ ]$ is a ternary relation on a set X interpreting a notion of betweenness, then we say the structure $\langle X, [\ ,\ ,\ ]\rangle$ is *gap free* if each two elements of X always have a third element between them. This amounts to satisfying the universal-existential sentence

$$\text{Gap Freeness: } \forall\, ab\ \exists\ x\ (a \neq b \rightarrow ([a, x, b] \wedge x \neq a \wedge x \neq b))$$

in the appropriate first-order language L_t (see, e.g., [5]).

This paper is a continuation of [1], in which road systems are introduced as a means of unifying the majority of known interpretations of the intuitive notion of betweenness. Briefly, a *road system* is an ordered pair $\langle X, \mathcal{R}\rangle$, where X is a nonempty set of points and $\mathcal{R}$ is a collection of nonempty subsets of X—the *roads*—satisfying (1) every singleton subset of X is a road and (2) every doubleton subset of X is contained in at least one road. The road system is *additive* if the union of two overlapping roads is a road; the system is *separative* if for any $a, b, c \in X$, with

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