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COUNTABLE IRRESOLVABLE SPACES AND CARDINAL INVARIANTS

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Abstract. Answering a question of Marion Scheepers, we show that the cardinal invariant $\mathfrak{d}$ is a lower bound on $\mathfrak{irr}$, the minimal weight (equivalently, minimal π-weight) of a countable regular irresolvable space. We consider related cardinal invariants such as $\mathfrak{r}_{\mathsf{scat}}$, the reaping number of the quotient algebra $\mathcal{P}(\mathbb{Q})$ mod the ideal of scattered subsets of the rationals, and prove that $\diamondsuit(\mathfrak{r}_{\mathsf{scat}})$ implies that $\mathfrak{irr} = \omega_1$.

1. Introduction

All topological spaces considered are regular and *crowded*, i.e., have no isolated points. A topological space X is said to be *irresolvable* provided there are no disjoint dense subsets Y, $W \subseteq X$. Otherwise, X is *resolvable.*

It is easy to see that $\mathbb{Q}$ is a resolvable space. It follows, due to a well-known theorem of Sierpiński, that every countable first countable crowded regular space is resolvable. So, if X is a countable regular irresolvable space, $w(X)$ should be uncountable. In fact, the same is true for countable regular spaces with countable π-weight.

Marion Scheepers [6] defines the *irresolvability number* as follows:

$$\mathfrak{irr} = \min\{\pi w((\omega, \tau)) : \tau \subseteq \mathcal{P}(\omega) \textit{ is an irresolvable } T_3 \textit{ topology on } \omega\}.$$

It is folklore that $\mathfrak{r} \leq \mathfrak{irr} \leq \mathfrak{i}$ (see [3], [6]), where $\mathfrak{r}$ denotes the *reaping number* (the minimal size of a *reaping* (or *unsplittable*) family, i.e., the

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