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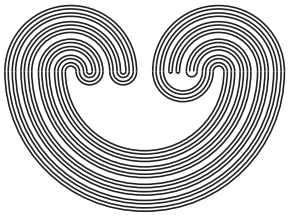

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Semicoverings, Coverings, Overlays, and Open Subgroups of the Quasitopological Fundamental Group

by

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SEMICOVERINGS, COVERINGS, OVERLAYS, AND OPEN SUBGROUPS OF THE QUASITOPOLOGICAL FUNDAMENTAL GROUP

JEREMY BRAZAS

Abstract. In this paper, we study the classification of semicovering maps, classical covering maps, and Ralph H. Fox's overlays in the context of open subgroups of (quasi)topological fundamental groups. For a given space X, we say a subgroup $H \subseteq \pi_1(X, x_0)$ is a semicovering (covering, overlay) subgroup if there is a semicovering (covering, overlay) $p : Y \to X$, $p(y_0) = x_0$ such that H is the image of the monomorphism induced on fundamental groups. Using a new type of Spanier group, we show that every overlay subgroup has open core (i.e., contains an open normal subgroup). We also use semicoverings to show that if X is a so-called locally wep-connected space, then every subgroup of $\pi_1(X, x_0)$ with open core is a covering subgroup. The converse holds for locally path connected spaces but not for general locally wep-connected spaces. We find application to the general theory of topological groups by identifying a large class of spaces Z whose free Graev topological group $F_G(Z, z)$ admits an open subgroup H with non-open core. This is achieved by constructing a covering map $p : E \to B$, which is not an overlay, similar to a well-known example of Fox.

1. Introduction

The covering spaces of a path connected, locally path connected, and semilocally simply connected topological space X are classified by the subgroups of the fundamental group $\pi_1(X, x_0)$. This classification is often stated in categorical terms: The categories of the coverings of X,

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Key words and phrases. covering map, overlay, quasitopological fundamental group, semicovering map.

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