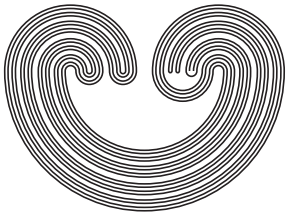
TOPOLOGY PROCEEDINGS



Volume 44, 2014

Pages 365–388

http://topology.auburn.edu/tp/

Quadratic Bounds on the Quasiconvexity of Nested Train Track Sequences

by

Tarik Aougab

Electronically published on March 4, 2014

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

http://topology.auburn.edu/tp/
TOPOLOGY PROCEEDINGS
Volume 44 (2014)
Pages 365-388
E-Published on March 4, 2014

QUADRATIC BOUNDS ON THE QUASICONVEXITY OF NESTED TRAIN TRACK SEQUENCES

TARIK AOUGAB

Abstract. Let $S_{g,p}$ denote the genus g orientable surface with p punctures. We show that nested train track sequences constitute $O((g,p)^2)$-quasiconvex subsets of the curve graph, effectivizing a theorem of Howard A. Masur and Yair N. Minsky. As a consequence, the genus g disk set is $O(g^2)$-quasiconvex. We also show that splitting and sliding sequences of birecurrent train tracks project to $O((g,p)^2)$-unparameterized quasigeodesics in the curve graph of any essential subsurface, an effective version of a theorem of Masur, Lee Mosher, and Saul Schleimer.

1. Introduction

Let $S_{g,p}$ denote the orientable surface of genus g with $p \geq 0$ punctures, and let $\mathcal{C}(S_{g,p})$ be the corresponding curve complex. Finally, let $\mathcal{C}_k(S_{g,p})$ denote the corresponding k-skeleton.

Let $(\tau_i)_i$ be a sequence of train tracks on $S_{g,p}$ such that τ_{i+1} is carried by τ_i for each i. Such a collection of train tracks defines a subset of $\mathcal{C}_0(S_{g,p})$ called a *nested train track sequence*. A *train track splitting sequence* is an important special case of such a sequence, in which τ_i is obtained from τ_{i-1} via one of two simple combinatorial moves, *splitting* and *sliding*.

A nested train track sequence is said to have R-bounded steps if the $\mathcal{C}_1$-distance between the vertex cycles of τ_i and those of τ_{i+1} is bounded above by R. Howard A. Masur and Yair N. Minsky [13] show that any

2010 *Mathematics Subject Classification.* 57M07, 20F65.
Key words and phrases. curve complex, disk set, mapping class roup.
The author was partially supported by an NSF grant during the completion of this work.