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TOPOLOGY PROCEEDINGS

Volume 45, 2015

Pages 121–138

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by

Simon Brandhorst and Marcel Erné

Electronically published on June 26, 2014

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

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TYCHONOFF-LIKE PRODUCT THEOREMS FOR LOCAL TOPOLOGICAL PROPERTIES

SIMON BRANDHORST AND MARCEL ERNÉ

Abstract. We consider classes $\mathcal{T}$ of topological spaces (referred to as $\mathcal{T}$-spaces) that are stable under continuous images and frequently under arbitrary products. A local $\mathcal{T}$-space has for each point a neighborhood base consisting of subsets that are $\mathcal{T}$-spaces in the induced topology. A general necessary and sufficient criterion for a product of topological spaces to be a local $\mathcal{T}$-space in terms of conditions on the factors enables one to establish a broad variety of theorems saying that a product of spaces has a certain local property (like local compactness, local sequential compactness, local σ-compactness, local connectedness etc.) if and only if each factor has that local property, almost all have the corresponding global property, and not too many factors fail a suitable additional condition. Many of the results admit a point-free formulation; a look at sum decompositions into components of spaces with local properties yields product decompositions into indecomposable factors for certain classes of frames like completely distributive lattices or hypercontinuous frames.

2010 *Mathematics Subject Classification.* Primary 54B10; Secondary 06B35, 54D05, 54D30, 54D45.

Key words and phrases. (locally) compact, (locally) connected, (local) $\mathcal{T}$-space, product space.