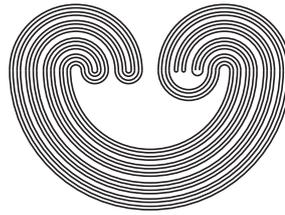


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THE ANTISYMMETRY BETWEENNESS AXIOM AND HAUSDORFF CONTINUA

by

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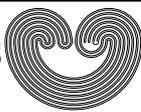
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THE ANTISYMMETRY BETWEENNESS AXIOM AND HAUSDORFF CONTINUA

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ABSTRACT. An interpretation of betweenness on a set satisfies the *antisymmetry* axiom at a point a if it is impossible for each of two distinct points to lie between the other and a . In this paper we study the role of antisymmetry as it applies to the *K-interpretation of betweenness* in a Hausdorff continuum X , where a point c lies between points a and b exactly when every subcontinuum of X containing both a and b contains c as well.

1. INTRODUCTION

An interpretation of betweenness on a set satisfies the *antisymmetry* axiom at a point a , or is *antisymmetric at a* , if it is impossible for each of two distinct points to lie between the other and a . Expressed as a first-order formula (see, e.g., [8]) involving just one ternary relation symbol and equality, this axiom is

Antisymmetry at a : $\forall xy (([a, y, x] \wedge [a, x, y]) \rightarrow x = y)$.

And from this it is clear that “antisymmetry at a ” is the usual order-theoretic notion of antisymmetry for the (generally reflexive and transitive) binary relation \leq_a , defined by saying $x \leq_a y$ exactly when x lies between a and y . Binary antisymmetry is the very condition that turns a pre-order into a partial order; we say that an interpretation of betweenness is *antisymmetric* if it is antisymmetric at each of its points.

2010 *Mathematics Subject Classification.* Primary 54F15; Secondary 03C05, 06A05, 06A06, 08A02, 08A40, 54F05, 54F20, 54F50, 54F55.

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