http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 45, 2015

Pages 189–215

http://topology.nipissingu.ca/tp/

The Antisymmetry Betweenness Axiom and Hausdorff Continua

by

Paul Bankston

Electronically published on October 9, 2014

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See http://topology.auburn.edu/tp/subscriptioninfo.html for information.

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 45 (2015)
Pages 189-215

E-Published on October 9, 2014

THE ANTISYMMETRY BETWEENNESS AXIOM AND HAUSDORFF CONTINUA

PAUL BANKSTON

Abstract. An interpretation of betweenness on a set satisfies the *antisymmetry* axiom at a point a if it is impossible for each of two distinct points to lie between the other and a. In this paper we study the role of antisymmetry as it applies to the K-*interpretation of betweenness* in a Hausdorff continuum X, where a point c lies *between* points a and b exactly when every subcontinuum of X containing both a and b contains c as well.

1. Introduction

An interpretation of betweenness on a set satisfies the *antisymmetry* axiom at a point a, or is *antisymmetric at* a, if it is impossible for each of two distinct points to lie between the other and a. Expressed as a first-order formula (see, e.g., [8]) involving just one ternary relation symbol and equality, this axiom is

$$\text{Antisymmetry at } a\text{: } \forall xy\,(([a,y,x] \wedge [a,x,y]) \to x = y).$$

And from this it is clear that "antisymmetry at a" is the usual order-theoretic notion of antisymmetry for the (generally reflexive and transitive) binary relation $\leq_a$, defined by saying $x \leq_a y$ exactly when x lies between a and y. Binary antisymmetry is the very condition that turns a pre-order into a partial order; we say that an interpretetion of betweenness is *antisymmetric* if it is antisymmetric at each of its points.

2010 *Mathematics Subject Classification.* Primary 54F15; Secondary 03C05, 06A05, 06A06, 08A02, 08A40, 54F05, 54F20, 54F50, 54F55.

Key words and phrases. Betweenness, interval, R-relation, road system, antisymmetry, gap freeness, centroid, median algebra, tree ordering, Hausdorff continuum, composant, hereditary unicoherence, decomposability, aposyndesis, arc, arboroid, dendron, distal continuum.

©2014 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See http://topology.auburn.edu/tp/subscriptioninfo.html for information.