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TOPOLOGY PROCEEDINGS

Volume 45, 2015

Pages 233–252

http://topology.nipissingu.ca/tp/

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by

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Electronically published on November 15, 2014

Topology Proceedings

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ISSN:	0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

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Volume 45 (2015)
Pages 233-252

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COMPLETELY ULTRAMETRIZABLE SPACES AND CONTINUOUS BIJECTIONS

W. R. BRIAN

Abstract. We say that two topological spaces are similar when each admits a continuous bijection onto the other. We will explore the similarity relation for spaces that can be represented as ω-length trees, namely the completely ultrametrizable spaces. We will prove that the separable, perfect, completely ultrametrizable spaces (i.e., the perfect zero-dimensional Polish spaces) come in exactly three similarity classes. The non-separable, perfect, completely ultrametrizable spaces are less tame, but we will show that, under the assumption of the Continuum Hypothesis, those of size $\mathfrak{c}$ come in exactly four similarity classes.

1. Introduction

This paper began as a short note about zero-dimensional Polish spaces and continuous bijections between them. The germ from which the paper grew is given below as Theorem 3.2: if X is a zero-dimensional Polish space and is not σ-compact, then there is a continuous bijection $X \to \mathcal{N}$ (here, as elsewhere, we use $\mathcal{N}$ to denote the Baire space ω^ω). This result, while new, has the same flavor as several classical results. For instance, it is known that every Polish space is the continuous image of $\mathcal{N}$, and that every perfect Polish space is the image of $\mathcal{N}$ under a continuous bijection. Our result shows that these properties are not unique to $\mathcal{N}$.

Every zero-dimensional Polish space can be represented as a tree, in a sense to be made precise in Section 2. The aforementioned results have proofs with a partly combinatorial flavor, with these trees playing a prominent part. One aim of this paper is to determine how well these proof techniques can be extended beyond the realm of zero-dimensional Polish spaces.

2010 *Mathematics Subject Classification.* Primary 54E99, 54C10, 03E35; Secondary 54H05.

Key words and phrases. Complete ultrametric, tree, condensation, Continuum Hypothesis, Polish space, Borel set.