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Electronically published on November 30, 2014

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ON ONE-LOCAL RETRACT IN QUASI-METRIC SPACES

OLIVIER OLELA OTAFUDU

ABSTRACT. We study a concept of 1-local retract in quasi-metric spaces. In this article, we generalize further known results about 1-local retract subsets from metric setting to quasi-metric point of view. In particular we show that any commuting family of nonexpansive self-mappings in a nonempty $T_0$-quasi-metric space $(X, d)$ for which $A_q(X)$ is compact and normal has a common fixed point and the common fixed point set is a 1-local retract of $(X, d)$.

1. Introduction

A subset $A$ of a metric space $(X, m)$ is said to be a 1-local retract of $(X, m)$ if for every family $\{B_i; i \in I\}$ of closed balls centered in $A$ with nonempty intersection, then $A \cap (\bigcap_{i \in I} B_i) \neq \emptyset$ (see [3], compare [4]). In [4], Khamsi showed that any commutative family of nonexpansive self-mapping defined on a metric space with compact and normal convexity structure has a common fixed point. In this article, we study the concept of 1-local retract in asymmetric setting. Among other things in this paper we consider subspaces 1-local retracts of a nonempty $T_0$-quasi-metric space and also present some fixed point theorems. In particular we prove that a nonexpansive self-mapping nonempty $T_0$-quasi-metric space $(X, d)$ for which the set of all $q$-admissible subsets of $X$ is compact and normal has at least one fixed point.

The concept of 1-local retract is due to Pouzet [4, p.4] and it has been investigated in detail by Khamsi and others (see for instance [3] and [4]). Our investigations are done in parallel with the well-known metric theory of 1-local retract (see [4]) and they confirm the surprising fact that many classical results about 1-local retract in metric spaces do not make essential use of the symmetry of the metric and thus still hold in quasi-metric spaces.

2010 Mathematics Subject Classification. 54E15, 54E35, 54C15,47H10.
Key words and phrases. Point, Normal structure, 1-local retract, $q$-admissible.
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