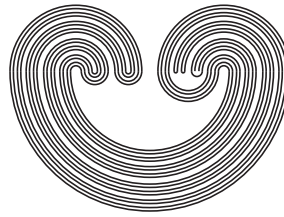


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## LOCALLY CONVEX $S$ -COMPACTIFICATIONS

by

H. BOUSTIQUE, B. LOSERT, AND G. RICHARDSON

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## LOCALLY CONVEX $S$ -COMPACTIFICATIONS

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**ABSTRACT.** Properties of a convergence semigroup  $S$  acting continuously on a locally convex convergence space  $X$  are investigated. Sufficient conditions are given for which the action can be continuously extended to a locally convex compactification of  $X$ . Local convexity is also extended to the generalized quotient of  $X$  and  $S$ .

### 1. INTRODUCTION AND PRELIMINARIES

Several articles have been written on the topic of a topological group or semigroup acting on a topological space. The authors have studied this notion in the larger category of convergence spaces. The category of convergence spaces has more desirable properties than the category of topological spaces. For example, quotient maps are both productive and hereditary in the category of convergence spaces, and these properties are particularly useful when forming the generalized quotient space. The primary focus of this work is to give sufficient conditions which guarantee that the action of a convergence monoid on a locally convex convergence space can be continuously extended to a locally convex compactification. Without convexity, this problem was investigated in [8]. It is shown in [7] that each locally convex convergence space possesses a locally convex compactification (no action considered). Basic results pertaining to a convergence monoid acting continuously on a convergence space (without convexity) can be found in [2] and [3].

Let  $X$  be a set, let  $\mathbf{P}(X)$  be the power set of  $X$ , and let  $\mathbf{F}(X)$  be the set of all filters on  $X$ . For each  $x \in X$ ,  $\dot{x}$  denotes the fixed ultrafilter on  $X$  whose base is  $\{\{x\}\}$ . Define a partial order  $\geq$  on  $\mathbf{F}(X)$  as follows:

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