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TOPOLOGY PROCEEDINGS



Volume 46, 2015

Pages 255–276

<http://topology.nipissingu.ca/tp/>

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Electronically published on July 31, 2014

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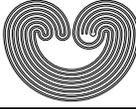
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ISSN: 0146-4124

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ABSTRACT. The “weakly Hausdorff” property for pseudoradial spaces fails to be naturally characterized by unique convergence of transfinite sequences. In response, we develop the category **SPsRad** of strongly pseudoradial spaces, compactly generated spaces whose closed sets are determined by globally continuous maps from well-ordered spaces. Categorically **SPsRad** is the coreflective hull of the class of well-ordered spaces, and **SPsRad** is Cartesian closed. The strongly pseudoradial weakly Hausdorff spaces admit a natural characterization involving unique extensions of injective maps of well-ordered spaces. We also obtain analogs in **SPsRad** of the fact that for sequential spaces, sequential compactness is equivalent to countable compactness.

1. INTRODUCTION

This paper introduces a category **SPsRad** of strongly pseudoradial spaces, a natural generalization of sequential spaces. The space X is *strongly pseudoradial* if, for each non-closed set $A \subset X$, there exists (with the order topology), a noncompact well-ordered space α and a map $f : \alpha \cup \{\infty_\alpha\} \rightarrow X$ so that $f(\alpha) \subset A$ and $f(\infty_\alpha) \notin A$. Here, $\alpha \cup \{\infty_\alpha\}$ denotes the familiar one point compactification of α .

Our motivation is the observation that the following fact about sequential spaces does not (as shown in Example 5.3) generalize naturally to pseudoradial spaces. If X is a sequential space, then each convergent sequence in X has a unique limit if and only if each compact subspace of

2010 *Mathematics Subject Classification.* Primary 54X10, 58Y30, 18D35; Secondary 55Z10.

Key words and phrases. Cartesian closed category, pseudoradial space, sequential space, strongly pseudoradial spaces.

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