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STRONGLY PSEUDORADIAL SPACES

JEREMY BRAZAS AND PAUL FABEL

Abstract. The "weakly Hausdorff" property for pseudoradial spaces fails to be naturally characterized by unique convergence of transfinite sequences. In response, we develop the category **SPsRad** of strongly pseudoradial spaces, compactly generated spaces whose closed sets are determined by globally continuous maps from well-ordered spaces. Categorically **SPsRad** is the coreflective hull of the class of well-ordered spaces, and **SPsRad** is Cartesian closed. The strongly pseudoradial weakly Hausdorff spaces admit a natural characterization involving unique extensions of injective maps of well-ordered spaces. We also obtain analogs in **SPsRad** of the fact that for sequential spaces, sequential compactness is equivalent to countable compactness.

1. Introduction

This paper introduces a category **SPsRad** of strongly pseudoradial spaces, a natural generalization of sequential spaces. The space X is *strongly pseudoradial* if, for each non-closed set $A \subset X$, there exists (with the order topology), a noncompact well-ordered space α and a map $f : \alpha \cup \{\infty_\alpha\} \to X$ so that $f(\alpha) \subset A$ and $f(\infty_\alpha) \notin A$. Here, $\alpha \cup \{\infty_\alpha\}$ denotes the familiar one point compactification of α.

Our motivation is the observation that the following fact about sequential spaces does not (as shown in Example 5.3) generalize naturally to pseudoradial spaces. If X is a sequential space, then each convergent sequence in X has a unique limit if and only if each compact subspace of

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