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DENSE SADDLES IN TORUS MAPS

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ABSTRACT. In this paper, we look at a specific class of maps in the torus and explore the consequences of this map having a dense set of periodic saddles. The main result states that under these assumptions, the torus splits into a countable number of invariant cylinders with disjoint interiors and the map is transitive on each cylinder.

1. Introduction

We will focus on a class of maps $F : \mathbb{T}^2 \to \mathbb{T}^2$ which are of the form

$$F : (x, y) = (mx, g(x, y))(\mod 1), \tag{1.1}$$

where $m \in \mathbb{N}$ is > 1 and $g : \mathbb{T}^2 \to S^1$ is C^2. The motivation of this work is to explore the connection between transitivity of a map and the existence of dense periodic saddles in the torus. There are maps on the torus which have dense periodic saddles but are not transitive, as shown at the end of this section. Our main result states that a map on the torus with dense saddles may not be transitive, but there will be a decomposition of the torus into a finite number of cylinders with disjoint interiors with the map transitive on each component.

Our approach will be by using an invariant structure in the tangent bundle called "*invariant, expanding cone system*", explained in Section 2.2. Cone-systems have been studied previously as geometric structures in vector bundles, for example in [4]. The reason we assume that our map has the form (1.1) is because it has an invariant expanding cone system.

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