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by

Collins Amburo Agyingi

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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EXTENSIONS OF ULTRAMETRIC SPACES

COLLINS AMBURO AGYINGI

Abstract. The concept of the tight span of a metric space was introduced and studied by Dress. It is known that his (Dress) theory is equivalent to the theory of the injective hull of a metric space independently discussed by Isbell some years earlier. Dress showed in particular that for a metric space X the tight extension T_X is maximal among the tight extensions of X. In a paper by Bayod et al., it was shown that Isbell's approach can be modified to work similarly for ultrametric spaces. They went ahead and constructed the tight extension for an arbitrary ultrametric space X, which in this article we shall call the ultrametric tight (*um*-tight) extension of X and is denoted uT_X. Continuing that work we show in the present paper that large parts of the theory developed by Dress do not use the triangle inequality of the metric and when appropriately modified will hold unchanged for ultrametric spaces. In particular we shall show that for an ultrametric space X, uT_X is a maximal (among the *um*-tight) extensions of X.

1. Introduction

We say that a metric space Y is "injective" if every mapping which increases no distance from a subspace of any metric space X to Y can be extended, increasing no distance, over X. These spaces were introduced in [2] by Aronszajn and Panitchpakdi, and they called them "hyperconvex."

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