http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 47, 2016

Pages 221–259

http://topology.nipissingu.ca/tp/

On the submetrizability number and i-weight of quasi-uniform spaces and paratopological groups

by

Taras Banakh and Alex Ravsky

Electronically published on October 17, 2015

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See http://topology.auburn.edu/tp/subscriptioninfo.html for information.

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 47 (2016)
Pages 221-259

E-Published on October 17, 2015

ON THE SUBMETRIZABILITY NUMBER AND i-WEIGHT OF QUASI-UNIFORM SPACES AND PARATOPOLOGICAL GROUPS

TARAS BANAKH AND ALEX RAVSKY

Abstract. We derive many upper bounds on the submetrizability number and i-weight of paratopological groups and topological monoids with open shifts. In particular, we prove that each first countable Hausdorff paratopological group is submetrizable thus answering a problem of Arhangelskii posed in 2002. Also we construct an example of a zero-dimensional (and hence regular) Hausdorff paratopological abelian group G with countable pseudocharacter which is not submetrizable. In fact, all results on the i-weight and submetrizability are derived from more general results concerning normally quasi-uniformizable and bi-quasi-uniformizable spaces.

Introduction

This paper was motivated by the following problem of Arhangelskii [1, 3.11] (also repeated by Tkachenko in his survey [26, 2.1]): *Does every first countable Hausdorff paratopological group admit a weaker metrizable topology?* A surprisingly simple answer to this problem was given by the authors in [4]. We just observed that each Hausdorff paratopological group G carries a natural uniformity generated by the base consisting of entourages $\{(x,y) \in G \times G : y \in UxU^{-1} \cap U^{-1}xU\}$ where U runs over open neighborhoods of the unit e in G. In [4] this uniformity was called the *quasi-Roelcke uniformity* on G and denoted by $\mathcal{Q}$. If G is first-countable, then the quasi-Roelcke uniformity $\mathcal{Q}$ is metrizable, which implies that the space G is submetrizable. Moreover, if the quasi-Roelcke uniformity $\mathcal{Q}$ is

2010 *Mathematics Subject Classification.* 54D10, 54D15, 54E15, 22A30.

Key words and phrases. Submetrizable space, i-weight, pre-uniformity, quasi-uniformity, paratopological group, topological monoid.

The first author has been partially financed by NCN grant DEC-2012/07/D/ST1/02087.

©2015 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See http://topology.auburn.edu/tp/subscriptioninfo.html for information.