http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 47, 2016

Pages 279–296

http://topology.nipissingu.ca/tp/

Neight: the nested weight of a topological space

by

William R. Brian

Electronically published on October 29, 2015

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 47 (2016)
Pages 279-296

E-Published on October 29, 2015

NEIGHT: THE NESTED WEIGHT OF A TOPOLOGICAL SPACE

WILLIAM R. BRIAN

Abstract. The neight (nested weight) of a topological space X is the smallest number of nests in X whose union provides a subbasis for X. We explore some basic properties of this function, emphasizing the connections of neight with the small inductive dimension, weight, character, and density of a space.

1. Definitions and Preliminaries

Let X be a topological space. A set $\mathcal{N}$ of subsets of X is called a **nest** if $\mathcal{N}$ is totally ordered by $\subseteq$. In this paper we consider the question: *How many nests does it take to generate a given topology?*

For example, if X is a LOTS (Linearly Ordered Topological Space with the order topology) then

$$\mathcal{N}_L = \{(-\infty, a) \colon a \in X\} \quad \text{and} \quad \mathcal{N}_R = \{(a, \infty) \colon a \in X\}$$

are two nests in X and $\mathcal{N}_L \cup \mathcal{N}_R$ provides a subbasis for X. A stronger result, given as Theorem 2.2 in [2], is that (for T_1 spaces) the topology of X is generated by two nests if and only if X is a GO space (recall that a GO space, or Generalized Order space, is any space that is homeomorphic to a subspace of a LOTS).

It is trivially true that every topological space X has a subbasis that can be written as a union of nests: for each open subset U of X, $\{U\}$ is

2010 *Mathematics Subject Classification.* Primary 54A25, 54D70; Secondary 54A20, 54F05, 54F45.

Key words and phrases. Neight, nest, halfdirection, dimension.