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by

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SELECTIVE STRONG SCREENABILITY AND A GAME

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Abstract. Selective versions of screenability and of strong screenability coincide in a large class of spaces. We show that the corresponding games are not equivalent in even such standard metric spaces as the closed unit interval. We identify sufficient conditions for ONE to have a winning strategy (Theorem 3.3), and necessary conditions for TWO to have a winning strategy (Theorem 4.6) in the selective strong screenability game.

1. Introduction

Unless specified otherwise, all topological spaces in this paper are assumed to be infinite. A collection $\mathcal{A}$ of subsets of a topological space (X,τ) is *discrete* if there is for each $x \in X$ a neighborhood U of x such that $|\{A \in \mathcal{A} : A \cap U \neq \emptyset\}| \leq 1$. Note that a finite family of nonempty sets whose closures are disjoint is a discrete family. An infinite family of sets with pairwise disjoint closures need not be discrete, as illustrated by the family $\{[\frac{1}{2n+1}, \frac{1}{2n}] : n \in \mathbb{N}\}$ of disjoint closed subsets of the real line. A disjoint family of open sets covering a space is automatically a discrete family of open sets.

A family $\mathcal{A}$ of sets *refines* a family $\mathcal{B}$ of sets if there is for each $A \in \mathcal{A}$ a $B \in \mathcal{B}$ such that $A \subseteq B$. The symbol $\mathcal{O}$ denotes the collection of all open covers of the space (X,τ). When Y is a subset of X, then $\mathcal{O}_Y$ denotes the set of covers of Y by sets open in X.

R.H. Bing introduced the notions of *screenable* and *strongly screenable* in [8]. A topological space (X,τ) is *strongly screenable* if there is for each open cover $\mathcal{U}$ of X a sequence $(\mathcal{V}_n : n < \omega)$ such that each $\mathcal{V}_n$ is a *discrete* collection of sets, each $\mathcal{V}_n$ refines $\mathcal{U}$, and $\bigcup\{\mathcal{V}_n : n < \omega\}$ is an open cover of X.

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