http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 47, 2016

Pages 313–329

http://topology.nipissingu.ca/tp/

Cardinality of regular spaces admitting only constant continuous functions

by

Krzysztof Chris Ciesielski and Jerzy Wojciechowski

Electronically published on November 25, 2015

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 47 (2016)
Pages 313-329

E-Published on November 25, 2015

CARDINALITY OF REGULAR SPACES ADMITTING ONLY CONSTANT CONTINUOUS FUNCTIONS

KRZYSZTOF CHRIS CIESIELSKI AND JERZY WOJCIECHOWSKI

Abstract. We show that an infinite cardinal number κ is the cardinality of some connected regular topological space X if and only if $\kappa \geq \omega_1$; such X can be separable if and only if $\omega_1 \leq \kappa \leq 2^{\mathfrak{c}}$; X can be both first countable and separable if and only if $\omega_1 \leq \kappa \leq \mathfrak{c}$; and X can be first countable if and only if $\kappa \geq \omega_1$. The main tools used in our investigation come from the analysis of several popular constructions of a regular topological space which is not completely regular. In particular, this work contains a concise and self-contained presentation of the examples of Mysior [8], Thomas [11], and those that can be constructed by the, so called, Jones' counterexample machine [6] (compare [10, pp. 27-28]). Our exposition is based on extracting a common core of these constructions.

We describe one of the simplest examples of a regular space which is not completely regular. It is of the first uncountable cardinality ω_1. We show that this example can be modified, by a variation of a construction of Herrlich [5], to a regular space Y of the same cardinality such that any continuous function from Y into any Hausdorff space Z with a countable pseudo-character is constant. Since this includes the case of $Z = [0, 1]$, Y is connected and not completely regular. Such space Y can be separable. Moreover, if we are interested only in $Z = \mathbb{R}$, then Y can be also first countable.

2010 *Mathematics Subject Classification.* Primary 54G15, 54A25; Secondary: 54C05.

Key words and phrases. Cardinal invariants, regularity, continuity, separability.