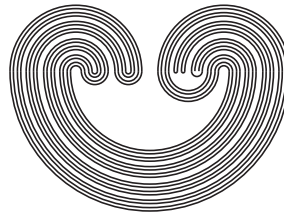


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CARDINALITY OF REGULAR SPACES ADMITTING ONLY CONSTANT CONTINUOUS FUNCTIONS

by

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ABSTRACT. We show that an infinite cardinal number κ is the cardinality of some connected regular topological space X if and only if $\kappa \geq \omega_1$; such X can be separable if and only if $\omega_1 \leq \kappa \leq 2^{\mathfrak{c}}$; X can be both first countable and separable if and only if $\omega_1 \leq \kappa \leq \mathfrak{c}$; and X can be first countable if and only if $\kappa \geq \omega_1$. The main tools used in our investigation come from the analysis of several popular constructions of a regular topological space which is not completely regular. In particular, this work contains a concise and self-contained presentation of the examples of Mysior [8], Thomas [11], and those that can be constructed by the, so called, Jones' counterexample machine [6] (compare [10, pp. 27-28]). Our exposition is based on extracting a common core of these constructions.

We describe one of the simplest examples of a regular space which is not completely regular. It is of the first uncountable cardinality ω_1 . We show that this example can be modified, by a variation of a construction of Herrlich [5], to a regular space Y of the same cardinality such that any continuous function from Y into any Hausdorff space Z with a countable pseudo-character is constant. Since this includes the case of $Z = [0, 1]$, Y is connected and not completely regular. Such space Y can be separable. Moreover, if we are interested only in $Z = \mathbb{R}$, then Y can be also first countable.

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