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ON HOMOGENEITY AND THE H-CLOSED PROPERTY

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Abstract. We establish several results concerning topological homogeneity and the weakening of compactness known as the H-closed property. First, it is shown that every Hausdorff space can be embedded in a homogeneous space that is the countable union of H-closed spaces. Second, it is shown that if X is an H-closed Urysohn homogeneous space then for every H-set set $A \subseteq X$, $x \in A$, and $y \notin A$, there exists a homeomorphism $h : X \to X$ such that $h(y) \in A$ and $h(x) \notin A$. This is an extension of Motorov's result that every compact homogeneous space is 1.5-homogeneous. Third, we show that the cardinality bound $2^{t(X)}$, shown to hold for a compact homogeneous space X by De La Vega, does not hold in general for H-closed homogeneous spaces. Last, we show the Katětov H-closed extension κX is never homogeneous if X is non-H-closed, and the remainder $\sigma X \backslash X$ in the H-closed Fomin extension σX is never power homogeneous if X is locally H-closed.

1. Introduction

A space X is *homogeneous* if for every $x, y \in X$ there exists a homeomorphism $h : X \to X$ such that $h(x) = y$. X is *power homogeneous* if there exists a cardinal κ such that X^κ is homogeneous. Many intriguing results have been obtained in the theory of compact homogeneous spaces; for example, De la Vega [11] showed that the cardinality of such a space X is at most $2^{t(X)}$, where $t(X)$ is the tightness of X. Motorov showed that a compact homogeneous space has a stronger form of homogeneity known as 1½-homogeneity (see [2]). Many deep questions concerning these spaces are still open (see, for example Jan van Mill's survey in [17]).

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