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GEODESIC CURVES ON SHIMURA SURFACES

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Dedicated to the memory of Colin Maclachlan

ABSTRACT. We parametrize the commensurability classes of curves on Shimura surfaces that are totally geodesic, i.e., the commensurability classes of so-called C-Fuchsian subgroups. In particular, if a Shimura surface contains one commensurability class of totally geodesic curves, it contains infinitely many.

1. INTRODUCTION

A Shimura surface is the quotient of either the product $\mathbf{H}^2 \times \mathbf{H}^2$ of two hyperbolic planes or the unit ball $\mathbf{H}_{\mathbb{C}}^2$ in \mathbb{C}^2 by an irreducible arithmetic lattice. Examples include the normal quasiprojective varieties associated with the Hilbert and Picard modular groups, along with the solutions to many moduli problems for principally polarized abelian varieties. Special amongst the immersed projective algebraic curves on these surfaces are those which are geodesic for the metric descending from the universal covering. In this paper, we completely classify the geodesic curves on Shimura surfaces up to commensurability. A consequence of this classification is the following.

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