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CATEGORICAL PROPERTIES ON THE HYPERSPACE OF NONTRIVIAL CONVERGENT SEQUENCES

S. GARCÍA-FERREIRA, R. ROJAS-HERNÁNDEZ, AND Y. F. ORTIZ-CASTILLO

Abstract. In this paper, we shall study categorial properties of the hyperspace of all nontrivial convergent sequences $S_c(X)$ of a Fréchet-Urysohn space $X$ equipped with the Vietoris topology. We mainly prove that $S_c(X)$ is meager whenever $X$ is a crowded space; as a corollary, we obtain that if $S_c(X)$ is Baire, then $X$ has a dense subset of isolated points. As an interesting example, $S_c(\omega_1)$ has the Baire property, where $\omega_1$ carries the order topology. (This answers a question from The hyperspace of convergent sequences, Topology Appl. 196 (2015), part B, 795–804.) We can give more examples like this one by proving that the Alexandroff duplicate $A(Z)$ of a space $Z$ satisfies that $S_c(A(Z))$ has the Baire property whenever $Z$ is a $\Sigma$-product of completely metrizable spaces and $Z$ is crowded. Also, we show that if $S_c(X)$ is pseudocompact, then $X$ has a relatively countably compact dense subset of isolated points, every finite power of $X$ is pseudocompact, and every $G_\delta$-point in $X$ must be isolated. We also establish several topological properties of the hyperspace of nontrivial convergent sequences of countable Fréchet-Urysohn spaces with only one non-isolated point.

1. Introduction

All our spaces will be Tychonoff (completely regular and Hausdorff). The letters $\mathbb{P}$ and $\mathbb{N}$ will denote the irrational numbers and the natural numbers, respectively. The positive natural numbers will be denoted by
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