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ON CUT POINTS PROPERTIES OF SUBCLASSES OF CONNECTED SPACES

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Abstract. We investigate, from the viewpoint of cut points, properties of some subclasses of connected spaces. We prove, without assuming any separation axiom, if a connected space $X$ has (*) i.e., $X$ has a closed $R(i)$ subset $H$ such that there is no proper regular closed, connected subset of $X$ containing $H$, then there is no proper connected subset of $X$ containing all non-cut points. This is used to show that a connected space having at most two non-cut points and (*) is a COTS with endpoints; also the converse holds. If, in addition, the space is locally connected, then it is compact. In an $R(i)$ connected space, each component of the complement of a cut point is found to contain a non-cut point of the space. For an $R(i)$ connected space $X$, it is also shown that if the removal of every two-point disconnected set leaves the space disconnected, then, for $a, b \in X$ and a separating set $H$ of $X – \{a,b\}$, $H \cup \{a,b\}$ is a $T_{2}^{*}$ $R(i)$ COTS with endpoints. Also we obtain some other characterizations of COTS with endpoints, and some characterizations of the closed unit interval.

1. Introduction

The idea of the concept of a cut point in a topological space dates back to 1920’s ([see 14, 15]). One of the reasons that the theory of cut points in topological spaces has been gaining importance is because it has found applications in computer science (see e.g. [8]). For the study of cut points, a topological space is assumed to be connected. Herein, by a space we mean a topological space. A point $x$ of a connected space $X$ is a cut point if $X – \{x\}$ is disconnected. So far, for the study of cut points, the space is assumed to be nondegenerate i.e., has at least two points. If a space contains only two points, then both points are non-cut points. We suppose that a space has at least three points.

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